

Homework 2

① $\int y^{-3} dy = \int \frac{1}{(t+1)^2} dt \rightsquigarrow -\frac{1}{2y^2} = \ln|t+1| + C$ ①

$$\frac{1}{y^2} = -2 \ln|t+1| + C$$

$$y(0) = -2 \ln(1) + C_1 = 1 \rightsquigarrow C_1 = 1$$

$$y(t) = \sqrt{1 - 2 \ln(t+1)}$$

② $z^2 - 3z + 2 = 0 \rightsquigarrow (z-2)(z-1) = 0 \rightsquigarrow z = 2, 1$

$$y(t) = C_1 e^{2t} + C_2 e^t$$

$$y'(t) = 2C_1 e^{2t} + C_2 e^t$$

$$y(0) = C_1 + C_2 = 2$$

$$y'(0) = 2C_1 + C_2 = 3$$

$$\left. \begin{array}{l} C_1 = 1 \\ C_2 = 1 \end{array} \right\}$$

$$y(t) = e^{2t} + e^t$$

③ $\det(A - \lambda I) = 0 \quad (3 - \lambda)^2 + 16 = 0 \rightsquigarrow 3 - \lambda = \pm 4i$

$$\lambda = 3 + 4i$$

$$\lambda = 3 \pm 4i$$

$$(A - \lambda I) = \begin{pmatrix} -4i & -4 \\ 4 & -4i \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-4ia - 4b = 0$$

$$4a - 4ib = 0$$

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\lambda = 3 - 4i$$

$$\begin{pmatrix} 4i & -4 \\ 4 & 4i \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$4ia - 4b = 0$$

$$4a - 4ib = 0$$

$$\vec{v}_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix} \text{ for } \lambda_1 = 3 + 4i, \quad \vec{v}_2 = \begin{pmatrix} 1 \\ i \end{pmatrix} \text{ for } \lambda_2 = 3 - 4i$$