Homework for Lecture 2 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file) to

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by 8:00pm Monday, Sept. 15, 2025.

Subject: hw2

with an attachment LastFirstHw2.pdf

1. Solve the following differential equation, subject to the given initial condition

$$\frac{dy}{dt} = \frac{y^3}{(t+1)} \quad , \quad y(0) = 1 \quad .$$

$\frac{dy}{y^3} = \frac{dt}{t+1} \Rightarrow \int y^{-3}\,dy = \int (t+1)^{-1}\,dt \Rightarrow -\frac{1}{2}y^{-2} = \ln(t+1) + C$

$\frac{-1}{2y^2} = \ln(t+1) - \frac{1}{2}$

$\frac{1}{y^2} = 1 - 2\ln(t+1)$

$y(t) = \frac{1}{\sqrt{1-2\ln(t+1)}}$

2. Solve the folloing differential equation, subject to the given initial conditions

$$y''(t) - 3y'(t) + 2y(t) = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

$r^2 - 3r + 2 = 0$

$(r-1)(r-2) \Rightarrow r = 1, 2$

$y(t) = C_1 e^t + C_2 e^{2t}$

$y'(t) = C_1 e^t + 2C_2 e^{2t}$

$C_1 + C_2 = 2$

$C_1 + 2C_2 = 3$

$\Rightarrow C_1 = 1, C_2 = 1$

$y(t) = e^t + e^{2t}$

3. Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

$\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} \rightarrow (3-\lambda)(3-\lambda) - (-16)$

$9 - 3\lambda - 3\lambda + \lambda^2 + 16$

$\lambda^2 - 6\lambda + 25$

$\frac{-b \pm \sqrt{b^2+4ac}}{2a}$

$\frac{6 \pm \sqrt{(-6)^2 - 4(1)(25)}}{2(1)}$

$\frac{6 \pm \sqrt{-64}}{2} \rightarrow \frac{6 \pm 8i}{2} = 3 \pm 4i$

$\sqrt{64}\,i$

$\lambda_1 = 3 + 4i$

$\begin{pmatrix} -4i & -4 \\ 4 & -4i \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow -4ix - 4y = 0 \Rightarrow y = -ix \cancel{6i}$

$x = 1 \Rightarrow v_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$

$v_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix} \quad v_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$