

3 - 4

4 3

$$1. \frac{dy}{dt} = \frac{y^3}{t+1}, \quad y(0) = 1$$

$$\int \frac{1}{y^3} dy = \int \frac{1}{t+1} dt$$

$$-\frac{1}{2}y^{-2} = \ln|t+1| + C$$

$$y^{-2} = -2\ln|t+1| + C$$

$$y = \frac{1}{\sqrt{-2\ln(t+1) + C}}$$

$$y(0) = \frac{1}{\sqrt{-2\ln(0+1) + C}} = 1$$

$$C = 1$$

$$y = \frac{1}{\sqrt{-2\ln(t+1) + 1}}$$

$$2. y'' - 3y' + 2y = 0$$

$$r^2 - 3r + 2$$

$$(r-2)(r-1)$$

$$r = 2, 1$$

$$y(0) = 2 \quad y'(0) = 3$$

$$y = c_1 e^x + c_2 e^{2x}$$

$$y(0) = c_1 + c_2 = 2$$

$$c_1, c_2 = 1$$

$$y' = c_1 e^x + 2c_2 e^{2x}$$

$$y'(0) = c_1 + 2c_2 = 3$$

$$y = e^x + e^{2x}$$



$$3. \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} Ax = \lambda x$$

$$(A - \lambda I)x = 0$$

$$A - \lambda I = \begin{pmatrix} 3 - \lambda & -4 \\ 4 & 3 - \lambda \end{pmatrix}$$

$$\det(A - \lambda I) = 9 - 6\lambda + \lambda^2 + 16$$

$$\lambda^2 - 6\lambda + 25 = 0$$

$$\frac{6 \pm \sqrt{36 - 100}}{2} = \frac{1}{2}(6 \pm \sqrt{-64}) = 3 \pm 4i$$

$$\lambda = 3 + 4i:$$

$$\begin{pmatrix} -4i & -4 \\ 4 & -4i \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-4ix_1 - 4x_2 = 0$$

$$4x_1 - 4ix_2 = 0$$

$$x_1 = ix_2$$

$$x_1 = i, x_2 = 1$$

$$\begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$\lambda = 3 - 4i:$$

$$\begin{pmatrix} 4i & -4 \\ 4 & 4i \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$4ix_1 - 4x_2 = 0$$

$$4x_1 + 4ix_2 = 0$$

$$ix_1 = x_2$$

$$x_1 = 1, x_2 = i$$

$$\begin{pmatrix} 1 \\ i \end{pmatrix}$$

Eigenvectors:

$$(i, 1) \text{ with } \lambda = 3 + 4i$$

$$(1, i) \text{ with } \lambda = 3 - 4i$$

Check:

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix} = \begin{pmatrix} 3i - 4 \\ 4i + 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 3 - 4i \\ 4 + 3i \end{pmatrix}$$

