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Homework #2

1.) $dy/dt = y^3/t+1$, $y(0)=1$

$y^{-3}dy = (t+1)dt$

$\Rightarrow \int y^{-3}dy = \int (t+1)dt$

$\Rightarrow -1/2y^2 = t^2/2 + t + C \Rightarrow -2y^2 = 1/t^2 + t + C$

gen. sol.: $y(t) = \sqrt{1/(-2t^2 - 2t + C)}$

$y(0) = 1 = \sqrt{1/(0 - 0 + C)} \Rightarrow C = 1$

IVP sol.: $y(t) = \sqrt{1/(-2t^2 - 2t + 1)}$

2.) $y''(t) - 3y'(t) + 2y(t) = 0$, $y(0)=2$, $y'(0)=3$

$r^2 - 3r + 2 = (r-2)(r-1) = 0 \Rightarrow r = 1, 2$

gen. sol.: $y(t) = C_1e^t + C_2e^{2t}$, $y_g'(t) = C_1e^t + 2C_2e^{2t}$

$y_g(0) = C_1 + C_2 = 2$

$y_g'(0) = C_1 + 2C_2 = 3$

$C_2 = 1$, $C_1 = 1$

IVP sol.: $y(t) = e^t + 2e^{2t}$

3.) $A = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$ $\quad |A - \lambda I| = \begin{vmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 + 16$

$= \lambda^2 - 6\lambda + 25 = 0$

$\lambda = \frac{6 \pm \sqrt{36 - 4(25)}}{2} = \frac{6 \pm \sqrt{-64}}{2} = 3 \pm 4i$

$\lambda_1 = 3+4i$, $\lambda_2 = 3-4i$

$[A - \lambda_1 I] = \begin{bmatrix} 3-3-4i & -4 \\ 4 & 3-3-4i \end{bmatrix} = \begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-4ia - 4b = 0$

$4a - 4ib = 0 \Rightarrow i(4a - 4ib) = 4ia + 4b = 0$

$ia = -b \quad ev_1 = \begin{bmatrix} i \\ -1 \end{bmatrix}$

$[A - \lambda_2 I] = \begin{bmatrix} 3-3+4i & -4 \\ 4 & 3-3+4i \end{bmatrix} = \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$4ia - 4b = 0 \quad \} \quad 4ia = 4b \Rightarrow ia = b$

$4a + 4ib = 0 \quad \}$

$ev_2 = \begin{bmatrix} i \\ 1 \end{bmatrix}$