Homework for Lecture 2 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file) to

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by 8:00pm Monday, Sept. 15, 2025.

Subject: hw2

with an attachment LastFirstHw2.pdf

1. Solve the following differential equation, subject to the given initial condition

$$\frac{dy}{dt} = \frac{y^3}{(t+1)} \quad , \quad y(0) = 1 \quad .$$

2. Solve the folloing differential equation, subject to the given initial conditions

$$y''(t) - 3y'(t) + 2y(t) \; = \; 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

3. Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

1. Solve the following differential equation, subject to the given initial condition

$$\frac{dy}{dt} = \frac{y^3}{(t+1)} \quad , \quad y(0) = 1 \quad .$$

$t=0, y=1$

$$\int y^{-3}\,dy = \int (t+1)^{-1}\,dt$$

$$-\tfrac{1}{2}y^{-2} = \ln|t+1| + C$$

$$-\tfrac{1}{2} = \ln|1| + C$$

$$-\tfrac{1}{2} = C$$

$$-\tfrac{1}{2}y^{-2} = \ln|t+1| - \tfrac{1}{2}$$

$$y^{-2} = -2\left(\ln|t+1| - \tfrac{1}{2}\right)$$

$$y = \sqrt{\frac{-1}{2\left(\ln|t+1| - \frac{1}{2}\right)}}$$

2. Solve the folloing differential equation, subject to the given initial conditions

$$y''(t) - 3y'(t) + 2y(t) = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

$$z^2 - 3z + 2 = 0$$

$$(z-2)(z-1) = 0$$

$$z = 2, 1$$

$$y(t) = C_1 e^{2t} + C_2 e^{t}$$

$$2 = C_1 + C_2$$

$$y'(t) = 2C_1 e^{2t} + C_2 e^{t}$$

$$3 = 2C_1 + C_2$$

$$3 = 2(2 - C_2) + C_2$$

$$3 = 4 - 2C_2 + C_2$$

$$-1 = -C_2$$

$$C_2 = 1 \quad C_1 = 1$$

$$y(t) = e^{2t} + e^{t}$$

3. Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix}$$

$$\det = (3-\lambda)(3-\lambda) + 16 = 0$$

$$9 - 6\lambda + \lambda^2 + 16 = 0$$

$$\lambda^2 - 6\lambda + 25 = 0$$

$$\lambda = \frac{6 \pm \sqrt{36 - 100}}{2}$$

$$\lambda = \frac{6 \pm \sqrt{-64}}{2}$$

$$\lambda = \frac{6 \pm 8i}{2}$$

$$\lambda = 3 \pm 4i$$

$$\lambda = 3+4i: \begin{bmatrix} 3-(3+4i) & -4 \\ 4 & 3-(3+4i) \end{bmatrix} = \begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$$-4ai - 4b = 0 \qquad b = -ai$$

$$\cancel{4a - 4bi = 0}$$

an eigenvector assoc. with $\lambda = 3+4i$ is $\begin{bmatrix} 1 \\ -i \end{bmatrix}$

$$\lambda = 3-4i: \begin{bmatrix} 3-(3-4i) & -4 \\ 4 & 3-(3-4i) \end{bmatrix} = \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$$4ai - 4b = 0$$

$$b = ai$$

an eigenvector assoc. with $\lambda = 3-4i$ is $\begin{bmatrix} 1 \\ i \end{bmatrix}$