

$$\textcircled{1} \quad \frac{dy}{dt} = \frac{y^3}{(t+1)} \quad y(0)=1$$

$$y^{-3} dy = (t+1)^{-1} dt$$

$$\int y^{-3} dy = \int (t+1)^{-1} dt \quad u = t+1$$

$$du = dt$$

$$-\frac{1}{2} \cdot y^{-2} = \ln|t+1| + C$$

$$y^{-2} = -2 \ln|t+1| + C$$

$$y = \frac{1}{\sqrt{-2 \ln|t+1| + C}}$$

$$1 = \frac{1}{\sqrt{C}} \quad C=1$$

$$y = \frac{1}{\sqrt{-2 \ln|t+1| + 1}}$$

$$\textcircled{a} \quad y(0)=1 \quad t \in \mathbb{R}$$

$$|-2 \ln|t+1|| \geq 0$$

$$\ln|t+1| < \frac{1}{2}$$

$$t < e^{1/2} - 1$$

$$1 < t < e^{1/2} - 1$$

$$\textcircled{2} \quad y''(t) - 3y'(t) + 2y(t) = 0 \quad y(0)=2 \quad y'(0)=3$$

$$r^2 - 3r + 2 = 0$$

$$C_1 e^t + C_2 e^{2t}$$

$$C_1 + C_2 = 2 \quad C_1 = 1$$

$$C_1 + 2C_2 = 3 \quad C_2 = 1$$

$$y(t) = e^t + e^{2t}$$

$$\textcircled{3} \quad \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} \rightarrow (3-\lambda)^2 + 16 = \lambda^2 - 6\lambda + 9 + 16$$

$$\lambda^2 - 6\lambda + 25$$

$$\lambda = \frac{6 \pm \sqrt{36-100}}{2} = 3 \pm 4i$$

$$\lambda = 3+4i$$

$$\begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$v_1$$

$$\lambda = 3-4i$$

$$\begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$v_2$$