9/15/25

Caroline Hill - Homework #2

1. $\frac{dy}{dt} = \frac{y^3}{(t+1)}$; $y(0) = 1$

$$\frac{1}{y^3}\,dy = \frac{1}{(t+1)}\,dt \rightarrow \int \frac{1}{y^3}\,dy = \int \frac{1}{(t+1)}\,dt$$

$$-\frac{1}{2y^2} + C = \ln|t+1| + C$$

aggregate constants

$$-\frac{1}{2y^2} = \ln|t+1| + C \;;\; y = \pm\sqrt{-\frac{1}{2(\ln|t+1| + C)}}$$

$$1 = \sqrt{-\frac{1}{2(\ln|0+1| + C)}} \;;\; C = -1/2$$

$$\boxed{y = \sqrt{-\frac{1}{2(\ln|t+1| - 1/2)}}}$$

2. $y''(t) - 3y'(t) + 2y(t) = 0$; $y(0) = 2$, $y'(0) = 3$

$y(t) = e^{\lambda t}$

characteristic polynomial: $\lambda^2 - 3\lambda + 2 = 0$; $(\lambda - 1)(\lambda - 2)$; $\lambda_1 = 1$, $\lambda_2 = 2$

$$\left.\begin{aligned} y(t) &= C_1 e^t + C_2 e^{2t} \;;\; y(0) = C_1 e^0 + C_2 e^{2(0)} \;;\; C_1 + C_2 = 2 \\ y'(t) &= C_1 e^t + 2C_2 e^{2t} \;;\; y'(0) = C_1 e^0 + 2C_2 e^{2(0)} \;;\; C_1 + 2C_2 = 3 \end{aligned}\right\} C_1 = 1,\ C_2 = 1$$

$$\boxed{y(t) = e^t + e^{2t}}$$

3. $A = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$

$$[A - \lambda I] = \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} \;;\; \det([A - \lambda I]) = (3-\lambda)^2 + 16 = 0 \;;\; \lambda^2 - 6\lambda + 25 = 0$$

through quadratic formula, $\lambda_1 = 3+4i$, $\lambda_2 = 3-4i$

$$e_{\lambda_1} = \begin{bmatrix} 3-(3+4i) & -4 \\ 4 & 3-(3+4i) \end{bmatrix} \left\} \begin{aligned} -4ix_1 - 4x_2 &= 0 \\ 4x_1 - 4ix_2 &= 0 \end{aligned} \right\} x_1 = i,\ x_2 = \text{free}$$

$$e_{\lambda_2} = \begin{bmatrix} 3-(3-4i) & -4 \\ 4 & 3-(3-4i) \end{bmatrix} \left\} \begin{aligned} 4ix_1 - 4x_2 &= 0 \\ 4x_1 + 4ix_2 &= 0 \end{aligned} \right\} x_1 = -i,\ x_2 = \text{free}$$

$$\boxed{\lambda_1 = 3+4i,\ e_{\lambda_1} = \begin{bmatrix} i \\ 1 \end{bmatrix} \text{ and } \lambda_2 = 3-4i,\ e_{\lambda_2} = \begin{bmatrix} -i \\ 1 \end{bmatrix}}$$