Homework for Lecture 2 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file) to

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by 8:00pm Monday, Sept. 15, 2025.

Subject: hw2

with an attachment LastFirstHw2.pdf

1. Solve the following differential equation, subject to the given initial condition

$$\frac{dy}{dt} = \frac{y^3}{(t+1)} \quad , \quad y(0) = 1 \quad .$$

2. Solve the folloing differential equation, subject to the given initial conditions

$$y''(t) - 3y'(t) + 2y(t) = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

3. Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

1. Solve the following differential equation, subject to the given initial condition

$$\frac{dy}{dt} = \frac{y^3}{(t+1)} \quad , \quad y(0) = 1 \quad .$$

$$\int y^{-3}\,dy = \int \frac{1}{(t+1)}\,dt$$

$$-\frac{y^{-2}}{2} = \ln|t+1| + C$$

use $y(\frac{1}{y^2} = -2\ln|t+1| + C$

use $y(0)=1$

$$\frac{1}{1} = -2\ln|1| + C$$

$$0 = 1$$

$$\frac{1}{y^2} = -2\ln|t+1| + 1$$

$$\frac{1}{\sqrt{-2\ln|t+1| + 1}} = y$$

2. Solve the folloing differential equation, subject to the given initial conditions

$$y''(t) - 3y'(t) + 2y(t) = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0$$

$$r = 1, 2$$

$$y(t) = c_1 e^t + c_2 e^{2t}$$

$$y(0) = 2 = c_1 + c_2 \rightarrow c_1 = 2 - c_2$$

$$y'(t) = c_1 e^t + 2c_2 e^{2t}$$

$$y'(0) = 3 = c_1 + 2c_2$$

$$3 = 2 + c_2$$

$$c_2 = 1$$

$$c_1 = 1$$

$$y(t) = e^t + e^{2t}$$

3. Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

$(3-\lambda)(3-\lambda)+16=0$

$9-6\lambda+\lambda^2+16=0$

$\lambda^2-6\lambda+25=0$

$\frac{6\pm\sqrt{36-4(1)(25)}}{2(1)} = \frac{6\pm 8i}{2} = 3\pm 4i$

$\lambda_1 = 3+4i$

$\begin{bmatrix} 3-(3+4i) & -4 \\ 4 & 3-(3+4i) \end{bmatrix}$

$\begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix}$

$-4ia-4b=0$

$-4ia=4b$

$-ia=b$

$\vec{v}_1 = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

$\lambda_2 = 3-4i$

$\begin{bmatrix} 3-(3-4i) & -4 \\ 4 & 3-(3-4i) \end{bmatrix}$

$\begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix}$

$4ia-4b=0$

$ia=b$

$\vec{v}_2 = \begin{bmatrix} 1 \\ i \end{bmatrix}$