Homework For Lecture 2: Ezra Chechik

1. $\dfrac{dy}{dt} = \dfrac{y^3}{(t+1)}$, $y(0)=1$

Seperable: $\displaystyle\int y^{-3}\,dy = \int \frac{1}{(t+1)}\,dt$

$\rightarrow \dfrac{-1}{2y^2} = \ln|t+1| + C$

arbitrary redefined

$\rightarrow \dfrac{1}{y^2} = -2\ln(t+1) + C$

$\rightarrow y = \sqrt{\dfrac{-1}{2\ln(t+1)+C}} \rightarrow 1 = \dfrac{-1}{0+C}$

$, C=-1 \rightarrow y_{\text{particular}} = \sqrt{\dfrac{-1}{2\ln(t+1)-1}}$

2. $y''(t) - 3y'(t) + 2y(t) = 0$, $y(0) = 2$, $y'(0) = 3$

$\rightarrow$ characteristic eq. $\lambda^2 - 3\lambda + 2 = 0$

$(\lambda-1)(\lambda-2) = 0$

$y(t)_{\text{general}} = C_1 e^{t} + C_2 e^{2t}$

$y'(t)_{\text{gen}} = C_1 e^{t} + 2C_2 e^{2t}$

IC: $2 = C_1 + C_2$; $3 = C_1 + 2C_2$

$C_1 = 1$, $C_2 = 1$

$y(t)_{\text{part}} = e^{t} + e^{2t}$

3. $\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$ Find all eigenvalues and corresponding eigenvectors.

$$\det(A-\lambda I)=0 \rightarrow \det\begin{pmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{pmatrix}=0$$

$$\rightarrow (3-\lambda)(3-\lambda)+16=0$$

$$\rightarrow \lambda^2-6\lambda+25=0$$

$$\lambda = \frac{6 \pm \sqrt{36-4(1)(25)}}{2} = 3 \pm \frac{\sqrt{-64}}{2}$$

$= 3 \pm 4i$ (two complex-valued eigenvalues)

To Find eigenvec : plug into $(A-\lambda I)$ and find Nul.

$\lambda = 3+4i$:

$$\rightarrow \left(\begin{array}{cc|c} 3-(3+4i) & -4 & 0 \\ 4 & 3-(3+4i) & 0 \end{array}\right) \leftrightarrow \left(\begin{array}{cc|c} -4i & -4 & 0 \\ 4 & -4i & 0 \end{array}\right)$$

$$\leftrightarrow \left(\begin{array}{cc|c} i & 1 & 0 \\ 1 & -i & 0 \end{array}\right) \leftrightarrow \left(\begin{array}{cc|c} 1 & -i & 0 \\ i & 1 & 0 \end{array}\right)$$

$$\leftrightarrow \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \left(\begin{array}{cc|c} 1 & -i & 0 \\ 0 & 0 & 0 \end{array}\right) \end{array} \quad - \begin{array}{l} x = iy \\ y = y \end{array} \quad y\begin{bmatrix} i \\ 1 \end{bmatrix}$$

an eigenvec is $\begin{bmatrix} i \\ 1 \end{bmatrix}$ (when $y=1$)

$\lambda = 3-4i$: $\left(\begin{array}{cc|c} 4i & -4 & 0 \\ 4 & 4i & 0 \end{array}\right) \leftrightarrow \left(\begin{array}{cc|c} i & -1 & 0 \\ 1 & i & 0 \end{array}\right)$

$\leftrightarrow \left(\begin{array}{cc|c} 1 & i & 0 \\ i & -1 & 0 \end{array}\right) \leftrightarrow \left(\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad x = -iy$

$y = y$

$y\begin{bmatrix} -i \\ 1 \end{bmatrix}$, an eigenvector is $\begin{bmatrix} i \\ -1 \end{bmatrix}$