Homework for Lecture 2 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file) to

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by 8:00pm Monday, Sept. 15, 2025.

Subject: hw2

with an attachment LastFirstHw2.pdf

1. Solve the following differential equation, subject to the given initial condition

$$\frac{dy}{dt} = \frac{y^3}{(t+1)} \quad , \quad y(0) = 1 \quad .$$

2. Solve the folloing differential equation, subject to the given initial conditions

$$y''(t) - 3y'(t) + 2y(t) \;=\; 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

3. Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

Daniyal Chaudhry HW 2

1. $\underbrace{\frac{dy}{dt} = \frac{y^3}{(t+1)}}$ $\quad y(0) = 1$

$\int dy \frac{1}{y^3} = \int dt \frac{1}{t+1} \Rightarrow -\frac{1}{2y^2} = \ln|t+1| + C \Rightarrow$ if $t=0,\ y=1$: $\quad -\frac{1}{2} = \ln(1) + C \quad : C = -\frac{1}{2}$

$\hookrightarrow \quad -\frac{1}{2y^2} = \ln(t+1) - \frac{1}{2} \Rightarrow \frac{1}{y^2} = 1 - 2\ln(t+1)$

$\Rightarrow y = \sqrt{\frac{1}{1-2\ln(t+1)}}$

2. $y'' - 3y' + 2y = 0 \quad y(0) = 2,\ y'(0) = 3$

$r^2 - 3r + 2 = 0 \rightarrow (r-2)(r-1) \rightarrow r = 1, 2$: $y(t)$: $c_1 e^t + c_2 e^{2t} \longrightarrow y(0) = 2 = c_1 + c_2$ $\quad c_2 = 1$

$y'(t)$: $c_1 e^t + 2c_2 e^{2t} \longrightarrow y'(0) = 3 = c_1 + 2c_2$ $\quad c_1 = 1$

THUS: $y(t) = e^t + e^{2t}$

3. $\det(A - \lambda I) \rightarrow \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix}$ $\quad 9 - 6\lambda + \lambda^2 + 16 \rightarrow \lambda^2 - 6\lambda + 25 \rightarrow \frac{6 \pm \sqrt{36-100}}{2} \rightarrow \lambda = 3 \pm 4i$

$\lambda_1 = 3+4i$: $\begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow a = ib \quad \begin{bmatrix} ib \\ b \end{bmatrix}$ for $b \in \mathbb{R}$

$\lambda_2 = 3-4i$: $\begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow a = -ib \quad \begin{bmatrix} -ib \\ b \end{bmatrix}$ for $b \in \mathbb{R}$

$\lambda_1 = 3+4i,\ \vec{v_1} = \begin{bmatrix} ib \\ b \end{bmatrix}\ b \in \mathbb{R}$

$\lambda_2 = 3-4i,\ \vec{v_2} = \begin{bmatrix} -ib \\ b \end{bmatrix}\ b \in \mathbb{R}$