1. $\frac{dy}{dt} = \frac{y^3}{t+1}$

$\frac{dy}{y^3} = \frac{dt}{t+1}$

$\int \frac{dy}{y^3} = \int \frac{dt}{t+1}$

$-\frac{1}{2y^2} = \ln(t+1) + C$

$\frac{1}{y^2} = -2\ln(t+1) + C$

$y^2 = \frac{1}{-2\ln(t+1) + C}$

$y = \frac{1}{\sqrt{-2\ln(t+1) + C}}$

$y(0) = 1 = \frac{1}{\sqrt{-2\ln(1) + C}}$

$1 = \frac{1}{\sqrt{C}} =$

$y = \frac{1}{\sqrt{1 - 2\ln(t+1)}}$

2 $y'' - 3y' + 2y = 0$

$c_1 e^{2x} + c_2 e^{x}$

$z^2 - 3z + 2 = 0$

$(z-2)(z-1) = 0$

$y(0) = 2 = c_1 e^{0} + c_2 e^{0}$

$y'(0) = 3 = 2c_1 e^{0} + c_2 e^{0}$

$y = e^{x} + e^{2x}$

$c_1 + c_2 = 2$

$2c_1 + c_2 = 3$

$2c_1 + 2c_2 = 4$

$2c_1 + c_2 = 3$

$c_2 = 1$

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3. $\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix}$

$9 - 6\lambda + \lambda^2 + 16$

$\lambda^2 - 6\lambda + 25 = 0$

$\frac{6 \pm \sqrt{36 - 100}}{2}$

$6 \pm \sqrt{-64}$

$\frac{6 \pm 8i}{2}$

eigenvalues: $3+4i$, $3-4i$

$\begin{bmatrix} 3-(3-4i) & -4 \\ 4 & 3-(3-4i) \end{bmatrix} = \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0$

$4ia - 4b = 0$

$4a + 4bi = 0$

$ai - b = 0$

$a + bi = 0$

2 cont

$$-a+bi=0 \qquad \begin{bmatrix} ai \\ a \end{bmatrix}$$
$$ai-b=0$$

$$ai=b$$

$$\begin{bmatrix} 3-(3+4i) & -4 \\ 4 & 3-(3+4i) \end{bmatrix} = \begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} =$$

$$\begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0$$

$$-4ai-4b=0 \qquad \begin{bmatrix} -ai \\ a \end{bmatrix}$$
$$4a-4bi=0$$
$$4ai+4b=0$$
$$4ai=-4b$$
$$ai=-b$$
$$a=bi$$

eigenvalues: $3+4i,\ 3-4i$

eigenvectors: $\begin{bmatrix} i \\ 1 \end{bmatrix}, \begin{bmatrix} -i \\ 1 \end{bmatrix}$