

1. $\frac{dy}{dt} = \frac{y^3}{(t+1)}$ HW2 $y(0)=1$

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$$\int y^{-3} dy = \int \frac{1}{t+1} dt$$

$$\frac{y^{-2}}{-2} = \ln(t+1) + C_1 \quad \text{where } C_1 \text{ is an arbitrary constant}$$

$$y^{-2} = -2 \ln(t+1) + C_2 \quad C_2 = -2C_1$$

$$y^2 = \frac{1}{-2 \ln(t+1) + C_2}$$

$$y(t) = \pm \sqrt{\frac{1}{C_2 - 2 \ln(t+1)}}$$

As $y(0) = 1 > 0$, $y(t)$ must be the positive expression.

$$y(0) = \sqrt{\frac{1}{C_2}} = 1$$

$$1 = \frac{1}{C_2} \Rightarrow C_2 = 1$$

$$y(t) = \sqrt{\frac{1}{1 - 2 \ln(t+1)}} = \frac{1}{\sqrt{1 - 2 \ln(t+1)}}$$

2. $y''(t) - 3y'(t) + 2y(t) = 0$, $y(0)=2$, $y'(0)=3$

Finding characteristic.

$$\begin{aligned} z^2 - 3z + 2 &= 0 \\ &= (z-2)(z-1) = 0 \\ &\Rightarrow z_1 = 1, z_2 = 2 \end{aligned}$$

$$\text{So } y(t) = A e^{t} + B e^{2t} \quad y(0) = 2 = A + B$$

$$y'(t) = A e^{t} + 2B e^{2t} \quad y'(0) = 3 = A + 2B$$

$$\Rightarrow (A + 2B) - (A + B) = 3 - 2$$

$$\Rightarrow B = 1$$

$$\Rightarrow A + 1 = 2$$

$$\Rightarrow A = 1, B = 1$$

~~$$\text{So } y(t) = e^t + e^{2t}$$~~

$$\text{So } y(t) = e^t + e^{2t}$$

3. $A = \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$

characteristic equation $\det(A - \lambda I) = 0$

$$\begin{vmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 - (-16) = 0$$

$$\Rightarrow (3-\lambda)^2 = -16$$

$$\Rightarrow 3-\lambda = \pm 4i$$

$$\Rightarrow \lambda = 3 \pm 4i$$

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Eigenvalues are: $\lambda_1 = 3 + 4i$ $\lambda_2 = 3 - 4i$
Let's solve $(A - \lambda_1 I) \vec{v}_1 = \vec{0}$

$$\begin{pmatrix} 3 - (3 + 4i) & -4 \\ 4 & 3 - (3 + 4i) \end{pmatrix} \begin{pmatrix} v_a \\ v_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -4i & -4 \\ 4 & -4i \end{pmatrix} \begin{pmatrix} v_a \\ v_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\text{So } -4i v_a - 4 v_b = 0 \\ \Rightarrow v_b = -i v_a$$

$$\text{and } 4 v_a - 4i v_b = 0$$

$$4 v_a - 4i(-i v_a) = 4 v_a - 4 v_a = 0, \text{ Verified.}$$

So we can have an eigenvector given by $v_b = -i v_a = 1$

$$\text{i.e. } \vec{v}_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Now let's solve $(A - \lambda_2 I) \vec{v}_2 = \vec{0}$

$$\begin{pmatrix} 3 - (3 - 4i) & -4 \\ 4 & 3 - (3 - 4i) \end{pmatrix} \begin{pmatrix} v_c \\ v_d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} +4i & -4 \\ 4 & +4i \end{pmatrix} \begin{pmatrix} v_c \\ v_d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$4i v_c - 4 v_d = 0 \Rightarrow v_d = i v_c$$

$$4 v_c + 4i v_d = 0$$

$$4 v_c + 4i(i v_c) = 4 v_c - 4 v_c = 0$$

So if we let $v_c = 1$, then $v_d = i$.

We get an eigenvector $\begin{pmatrix} 1 \\ i \end{pmatrix}$. So $\vec{v}_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$

I will clarify that $\vec{v}_1$ or $\vec{v}_2$ can be multiplied by any arbitrary constant and will still remain an eigenvector.