1. Solve the following differential equation, subject to the given initial condition

$$\frac{dy}{dt} = \frac{y^3}{(t+1)} \quad , \quad \overset{y(t)}{y(0) = 1} \quad .$$

$$\int \frac{dy}{y^3} = \int \frac{1}{t+1}\,dt$$

$$\frac{-1}{2y^2} = \ln|t+1| + C$$

plug in intial condition $y(0)=1$

$$\frac{-1}{2(1)^2} = \ln|1| + C$$

$$\frac{-1}{2} = C$$

$$-2\left(\frac{-1}{2y^2} = \ln(t+1) - \frac{1}{2}\right)$$

$$\frac{1}{y^2} = -2\ln(t+1) + 1$$

since $y(0) = 1 > 0$
take positive sq. root

$$\boxed{y(t) = \frac{1}{\sqrt{1 - 2\ln(t+1)}}}$$

Domain requires $t > -1$

$\hookrightarrow \ln(t+1)$ and $1 - 2\ln(t+1) \Rightarrow t < e^{1/2} - 1$

sol'n is only valid for $-1 < t < e^{1/2} - 1$

2. Solve the folloing differential equation, subject to the given initial conditions

$$y''(t) - 3y'(t) + 2y(t) = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

$r^2 - 3r + 2 = 0$

$(r-2)(r-1) = 0$

$r = 2, 1$

gen sol'n: $y(t) = C_1 e^{2t} + C_2 e^{t}$

$y(0) = 2 = C_1 + C_2$

$y'(t) = 2C_1 e^{2t} + C_2 e^{t}$

$y'(0) = 3 = 2C_1 e^{0} + C_2 e^{0}$

$C_1 + \cancel{C_2} = 2$

$-\ (2C_1 + \cancel{C_2} = 3)$

$C_1 = 1$

$C_2 = 1$

$$\boxed{y(t) = e^{2t} + e^{t}}$$

3. Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\det(A-\lambda I)=0 \qquad \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

$$\det\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix}=0$$

$$(3-\lambda)(3-\lambda)+16=0 \qquad \lambda = \frac{6\pm\sqrt{36-4(25)}}{2}$$

$$9-3\lambda-3\lambda+\lambda^2+16=0 \qquad \lambda=\frac{6\pm 8i}{2} \Rightarrow \lambda = 3\pm 4i$$

$$\lambda^2-6\lambda+25=0$$

If $\lambda = 3+4i$

$$\begin{bmatrix} 3-(3+4i) & -4 \\ 4 & 3-(3+4i) \end{bmatrix} = \begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} = \begin{bmatrix} i & 1 \\ 1 & -i \end{bmatrix}$$

$R_1/-4$
$R_2/4$

$$\begin{bmatrix} i & +1 \\ 1 & -i \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \boxed{\text{If } \lambda = 3+4i,\ \vec{v}=\begin{bmatrix} 1 \\ -i \end{bmatrix}}$$

$ai+b=0 \rightarrow b=-ai$ let $a=1$
$a-ib=0$ $b=-i$

If $\lambda = 3-4i$

$$\begin{bmatrix} 3-(3-4i) & -4 \\ 4 & 3-(3-4i) \end{bmatrix} = \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} = \begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}$$

$$\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \boxed{\text{If } \lambda = 3-4i \quad \vec{v}=\begin{bmatrix} 1 \\ i \end{bmatrix}}$$

$a=1$
$b=i$

$ai-b=0$
$a+bi=0$