Homework for Lecture 18 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Nov. 10, 2025.

Subject: hw18

with an attachment hw18FirstLast.pdf

1. In the (continuous) SIRS model with a population of 1000 and parameters $\gamma = 1.2$, $\nu = 1.2$. For each $\beta = 0.01 \cdot i$, for $1 \leq i \leq 20$, how many "removed" people are there?

2. Type

```
a1:=rand(1..100)():  a2:=rand(1..100)():[a1,a2];SEquP(ChemoStat(N,C,a1,a2),[N,C]);
```

20 times. How often did you get a stable equilibrium?

3. Run

```
SIRSdemo(1000,400,1,1,0.01,10);
```

4. After downloading

BOTH DMB.txt and L18.txt from the class web-page run

HWgE(100,1000);

10 times. Are the answers close to each other? Can you estimate the prob. that with a random preferance matrix only one genotype will survive in the long run?

1. In the (continuous) SIRS model with a population of 1000 and parameters $\gamma = 1.2$, $\nu = 1.2$. For each $\beta = 0.01 \cdot i$, for $1 \leq i \leq 20$, how many "removed" people are there?

SIRS(s, i, beta, gamma, nu, N);

s = susceptibles

i = # infected

N-s-i = removed

$\beta = .01 \cdot i$

i = 1 to 20

N = total pop

removed = N - s - i

i = 1

N = 1000

0 = 1000 - s - i

999

[s, i]

```
> SIRS(999, 1, .01, 1.2, 1.2, 1000);
                                [-9.99, 8.79]
> SIRS(998, 2, .02, 1.2, 1.2, 1000);
                                [-39.92, 37.52]
> SIRS(997, 3, .03, 1.2, 1.2, 1000);
                                [-89.73, 86.13]
> SIRS(996, 4, .04, 1.2, 1.2, 1000);
                                [-159.36, 154.56]
> SIRS(995, 5, .05, 1.2, 1.2, 1000);
                                [-248.75, 242.75]
> SIRS(994, 6, .06, 1.2, 1.2, 1000);
                                [-357.84, 350.64]
> SIRS(993, 7, .07, 1.2, 1.2, 1000);
                                [-486.57, 478.17]
> SIRS(992, 8, .08, 1.2, 1.2, 1000);
                                [-634.88, 625.28]
> SIRS(991, 9, .09, 1.2, 1.2, 1000);
                                [-802.71, 791.91]
> SIRS(990, 10, .1, 1.2, 1.2, 1000);
                                [-990.0, 978.0]
> SIRS(989, 11, .11, 1.2, 1.2, 1000);
                                [-1196.69, 1183.49]
> SIRS(988, 12, .12, 1.2, 1.2, 1000);
                                [-1422.72, 1408.32]
> SIRS(987, 13, .13, 1.2, 1.2, 1000);
                                [-1668.03, 1652.43]
> SIRS(986, 14, .14, 1.2, 1.2, 1000);
                                [-1932.56, 1915.76]
> SIRS(985, 15, .15, 1.2, 1.2, 1000);
                                [-2216.25, 2198.25]
> SIRS(984, 16, .16, 1.2, 1.2, 1000);
                                [-2519.04, 2499.84]
> SIRS(983, 17, .17, 1.2, 1.2, 1000);
                                [-2840.87, 2820.47]
> SIRS(982, 18, .18, 1.2, 1.2, 1000);
                                [-3181.68, 3160.08]
> SIRS(981, 19, .19, 1.2, 1.2, 1000);
                                [-3541.41, 3518.61]
> SIRS(980, 20, .2, 1.2, 1.2, 1000);
                                [-3920.0, 3896.0]
>
```

2. Type

```
a1:=rand(1..100)():  a2:=rand(1..100)():[a1,a2];SEquP(ChemoStat(N,C,a1,a2),[N,C]);
```

20 times. How often did you get a stable equilibrium? stable equilibrium 20 times

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[93, 45]$

$\{[4183.989130, 0.01086956522]\}$ (6)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[44, 100]$

$\{[4398.976744, 0.02325581395]\}$ (7)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[38, 69]$

$\{[2620.972973, 0.02702702703]\}$ (8)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[27, 96]$

$\{[2590.961538, 0.03846153846]\}$ (9)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[17, 90]$

$\{[1528.937500, 0.06250000000]\}$ (10)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[34, 18]$

$\{[610.9696970, 0.03030303030]\}$ (11)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[52, 56]$

$\{[2910.980392, 0.01960784314]\}$ (12)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[43, 83]$

$\{[3567.976190, 0.02380952381]\}$ (13)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[25, 90]$

$\{[2248.958333, 0.04166666667]\}$ (14)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[93, 60]$

$\{[5578.989130, 0.01086956522]\}$ (15)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[93, 14]$

$\{[1300.989130, 0.01086956522]\}$ (16)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[50, 47]$

$\{[2348.979592, 0.02040816327]\}$ (17)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[8, 46]$

$\{[366.8571429, 0.1428571429]\}$ (18)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[44, 9]$

$\{[394.9767442, 0.02325581395]\}$ (19)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[77, 59]$

$\{[4541.986842, 0.01315789474]\}$ (20)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[16, 1]$

$\{[14.93333333, 0.06666666667]\}$ (21)

>

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[70, 77]$

$\{[5388.985507, 0.01449275362]\}$ (22)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[39, 92]$

$\{[3586.973684, 0.02631578947]\}$ (23)

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$[71, 67]$

$\{[4755.985714, 0.01428571429]\}$ (24)

3. Run

```
SIRSdemo(1000,400,1,1,0.01,10);
```

```
> SIRSdemo(1000, 400, 1, 1, 0.01, 10);
```

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 0.01, *and letting it run until time t=*, 10
with population size, 1000, *and fixed parameters nu=*, 1, *and gamma=*, 1
*where we change beta from 0.2*nu/N to 4*nu/N*

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is, $\frac{1}{1000}$

We start with , 400, *infected individuals, 0 removed and hence*, 600, *susceptible*
We will show what happens once time is close to, 10

beta is , $\frac{1}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [999.6693512, 0.04464970605]], [9.99, [999.6721666, 0.04424784393]], [10.00, [999.6749582, 0.04384959883]], [10.01, [999.6777263, 0.04345493819]]]

beta is , $\frac{3}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [998.8686058, 0.2679278925]], [9.99, [998.8764375, 0.2660514879]], [10.00, [998.8842153, 0.2641882307]], [10.01, [998.8919395, 0.2623380288]]]

beta is , $\frac{1}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [995.5661036, 1.464972088]], [9.99, [995.5885005, 1.457614750]], [10.00, [995.6107835, 1.450294524]], [10.01, [995.6329532, 1.443011223]]]

beta is , $\frac{7}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [982.9907292, 6.871557578]], [9.99, [983.0448236, 6.850124744]], [10.00, [983.0987363, 6.828761355]], [10.01, [983.1524679, 6.807467168]]]

beta is , $\frac{9}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [944.9550913, 25.07830676]], [9.99, [945.0414764, 25.04080455]], [10.00, [945.1276722, 25.00337789]], [10.01, [945.2136792, 24.96602657]]]

beta is , $\frac{11}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [866.8575732, 64.57449614]], [9.99, [866.9275067, 64.54449698]], [10.00, [866.9972772, 64.51456141]], [10.01, [867.0668854, 64.48468924]]]

beta is , $\frac{13}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [764.2055840, 117.1693099]], [9.99, [764.2277964, 117.1616555]], [10.00, [764.2499053, 117.1540354]], [10.01, [764.2719113, 117.1464495]]]

beta is , $\frac{3}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [667.4467215, 166.2827762]], [9.99, [667.4446531, 166.2847218]], [10.00, [667.4425717, 166.2866623]], [10.01, [667.4404774, 166.2885977]]]

beta is , $\frac{17}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [588.7326192, 205.7837146]], [9.99, [588.7278789, 205.7854544]], [10.00, [588.7231678, 205.7871777]], [10.01, [588.7184858, 205.7888844]]]

beta is , $\frac{19}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [526.3391708, 236.9273141]], [9.99, [526.3371276, 236.9274194]], [10.00, [526.3351118, 236.9275155]], [10.01, [526.3331234, 236.9276024]]]

beta is , $\frac{21}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [476.0755911, 261.9954214]], [9.99, [476.0755589, 261.9947893]], [10.00, [476.0755398, 261.9941570]], [10.01, [476.0755336, 261.9935246]]]

beta is , $\frac{23}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [434.6820669, 282.6567829]], [9.99, [434.6827642, 282.6561293]], [10.00, [434.6834631, 282.6554802]], [10.01, [434.6841635, 282.6548356]]]

beta is , $\frac{5}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [399.9447066, 300.0130619]], [9.99, [399.9454130, 300.0126472]], [10.00, [399.9461153, 300.0122378]], [10.01, [399.9468135, 300.0118336]]]

beta is , $\frac{27}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [370.3498315, 314.8101124]], [9.99, [370.3503056, 314.8099378]], [10.00, [370.3507743, 314.8097672]], [10.01, [370.3512378, 314.8096006]]]

beta is , $\frac{29}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [344.8255549, 327.5761726]], [9.99, [344.8257953, 327.5761533]], [10.00, [344.8260313, 327.5761363]], [10.01, [344.8262630, 327.5761215]]]

beta is , $\frac{31}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [322.5856476, 338.7004341]], [9.99, [322.5857299, 338.7004867]], [10.00, [322.5858094, 338.7005401]], [10.01, [322.5858863, 338.7005943]]]

beta is , $\frac{33}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [303.0363872, 348.4783244]], [9.99, [303.0363869, 348.4783944]], [10.00, [303.0363852, 348.4784644]], [10.01, [303.0363821, 348.4785344]]]

beta is , $\frac{7}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [285.7191728, 357.1389626]], [9.99, [285.7191408, 357.1390237]], [10.00, [285.7191083, 357.1390844]], [10.01, [285.7190753, 357.1391447]]]

beta is , $\frac{37}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [270.2735369, 364.8628645]], [9.99, [270.2735002, 364.8629086]], [10.00, [270.2734634, 364.8629522]], [10.01, [270.2734267, 364.8629953]]]

beta is , $\frac{39}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [256.4121907, 371.7940209]], [9.99, [256.4121603, 371.7940490]], [10.00, [256.4121301, 371.7940766]], [10.01, [256.4121001, 371.7941037]]]

4. After downloading

BOTH DMB.txt and L18.txt from the class web-page run

HWgE(100,1000);

10 times. Are the answers close to each other? Can you estimate the prob. that with a random preferance matrix only one genotype will survive in the long run?

10 values

$HWgE(100, 1000);$

0.5500000000

$HWgE(100, 1000);$

0.5280000000

$HWgE(100, 1000);$

0.5710000000

> $HWgE(100, 1000);$

0.5510000000

> $HWgE(100, 1000);$

0.5810000000

> $HWgE(100, 1000);$

0.5640000000

>

> $HWgE(100, 1000);$

0.5500000000

> **HWgE(100, 1000)**

0.5280000000

> $HWgE(100, 1000);$

0.5710000000

> $HWgE(100, 1000);$

$HWgE(100, 1000);$

0.5480000000

The values are close to each other. The probability that with a randam preferance matrix only 1 genotype will survive in the long run is about 55%.