Homework for Lecture 18 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Nov. 10, 2025.

Subject: hw18

with an attachment hw18FirstLast.pdf

1. In the (continuous) SIRS model with a population of 1000 and parameters $\gamma = 1.2$, $\nu = 1.2$. For each $\beta = 0.01 \cdot i$, for $1 \leq i \leq 20$, how many "removed" people are there?

2. Type

```
a1:=rand(1..100)():  a2:=rand(1..100)():[a1,a2];SEquP(ChemoStat(N,C,a1,a2),[N,C]);
```

20 times. How often did you get a stable equilibrium?

3. Run

```
SIRSdemo(1000,400,1,1,0.01,10);
```

4. After downloading

BOTH DMB.txt and L18.txt from the class web-page run

HWgE(100,1000);

10 times. Are the answers close to each other? Can you estimate the prob. that with a random preferance matrix only one genotype will survive in the long run?

Problem 1

```
read `DMB.txt`
```

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help (ProcedureName); for example Help(Feig);

For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(1)**

```
read `L18.txt`
```

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help (ProcedureName); for example Help(Feig);

For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(2)**

```
Help(SIRS)
```

SIRS(s,i,beta,gamma,nu,N): The SIRS dynamical model with parameters beta,gamma, nu,N (see section 6.6 of Edelstein-Keshet), s is the number of

Susceptibles, i is the number of infected, (the number of removed is given by N-s-i). N is the total population. Try:

SIRS(s,i,beta,gamma,nu,N); **(3)**

$$\beta := 0.01 \quad (4)$$

```
SIRS(s, i, 0.01, 1.2, 1.2, 1000)
```

$$[-1.21\, s + 1198.8,\ 0.01\, s - 1.2] \quad (5)$$

```
#at equilibrium you can solve for s,i and then r.
s := (1.2 * 1000) / 0.01;      # S* = γN/β
  i := (1000 − s) / 2;         # I* = (N - S)/2
  R := N − s − i;              # R= I
```

$$s := 120000.0000$$

$$i := -59500.00000$$

$$R := -59500.00000 \quad (6)$$

```
# Parameters
N := 1000;
g := 1.2;
nu := 1.2;

# Loop over beta = 0.01*i for j = 1..20
for j from 1 to 20 do
  beta := 0.01 * j;
```

```
#at equilibrium you can solve for s,i and then r (set derivative equal to 0)
s := g * N / beta;
i := (N - s) / 2;
r := N - s - i;

# Run SIRS model
sol := SIRS(s, i, beta, g, nu, N);

# Print results
printf("beta = %.2f, s = %.2f, i= %.2f, r = %.2f\n", beta, s, i, r);
print(sol);
od:
```

$$N := 1000$$

$$g := 1.2$$

$$\nu := 1.2$$

```
beta = 0.01, s = 120000.00, i= -59500.00, r = -59500.00
```

$$[7.132860000 \times 10^7, -7.132860000 \times 10^7]$$

```
beta = 0.02, s = 60000.00, i= -29500.00, r = -29500.00
```

$$[3.536460000 \times 10^7, -3.536460000 \times 10^7]$$

```
beta = 0.03, s = 40000.00, i= -19500.00, r = -19500.00
```

$$[2.337660000 \times 10^7, -2.337660000 \times 10^7]$$

```
beta = 0.04, s = 30000.00, i= -14500.00, r = -14500.00
```

$$[1.738260000 \times 10^7, -1.738260000 \times 10^7]$$

```
beta = 0.05, s = 24000.00, i= -11500.00, r = -11500.00
```

$$[1.378620000 \times 10^7, -1.378620000 \times 10^7]$$

```
beta = 0.06, s = 20000.00, i= -9500.00, r = -9500.00
```

$$[1.138860000 \times 10^7, -1.138860000 \times 10^7]$$

```
beta = 0.07, s = 17142.86, i= -8071.43, r = -8071.43
```

$$[9.676028570 \times 10^6, -9.676028570 \times 10^6]$$

```
beta = 0.08, s = 15000.00, i= -7000.00, r = -7000.00
```

$$\left[8.391600000\times 10^{6}, -8.391600000\times 10^{6}\right]$$

beta = 0.09, s = 13333.33, i= -6166.67, r = -6166.67

$$\left[7.392599998\times 10^{6}, -7.392599998\times 10^{6}\right]$$

beta = 0.10, s = 12000.00, i= -5500.00, r = -5500.00

$$\left[6.593400000\times 10^{6}, -6.593400000\times 10^{6}\right]$$

beta = 0.11, s = 10909.09, i= -4954.55, r = -4954.55

$$\left[5.939509091\times 10^{6}, -5.939509091\times 10^{6}\right]$$

beta = 0.12, s = 10000.00, i= -4500.00, r = -4500.00

$$\left[5.394600000\times 10^{6}, -5.394600000\times 10^{6}\right]$$

beta = 0.13, s = 9230.77, i= -4115.38, r = -4115.38

$$\left[4.933523077\times 10^{6}, -4.933523077\times 10^{6}\right]$$

beta = 0.14, s = 8571.43, i= -3785.71, r = -3785.71

$$\left[4.538314286\times 10^{6}, -4.538314286\times 10^{6}\right]$$

beta = 0.15, s = 8000.00, i= -3500.00, r = -3500.00

$$\left[4.195800000\times 10^{6}, -4.195800000\times 10^{6}\right]$$

beta = 0.16, s = 7500.00, i= -3250.00, r = -3250.00

$$\left[3.896100000\times 10^{6}, -3.896100000\times 10^{6}\right]$$

beta = 0.17, s = 7058.82, i= -3029.41, r = -3029.41

$$\left[3.631658823\times 10^{6}, -3.631658823\times 10^{6}\right]$$

beta = 0.18, s = 6666.67, i= -2833.33, r = -2833.33

$$\left[3.396600001\times 10^{6}, -3.396600001\times 10^{6}\right]$$

beta = 0.19, s = 6315.79, i= -2657.89, r = -2657.89

$$\left[3.186284210\times 10^{6}, -3.186284210\times 10^{6}\right]$$

beta = 0.20, s = 6000.00, i= -2500.00, r = -2500.00

$$\left[2.997000000\times 10^{6}, -2.997000000\times 10^{6}\right] \quad \textbf{(7)}$$

#when beta<gamma → transmission rate <recovery rate), the infection dies out so no infected or removed people remain. Whne beta>gamma positive equilibrium with I>0 and R>0. Here the beta values go to .20 which are all much smaller than gamma (1.2) *thats why the removed people are negative counts. So techniqually, the removed amount of people should be* 0 **and if** *I were* **to** *increase* beta *we would see positive values of I* **and** *R.*

> **read**`DMB.txt`

#Problem 2

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig);
For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(1)**

> Help(ChemoStat)

ChemoStat(N,C,a1,a2): The Chemostat continuous-time dynamical system with N=Bacterial poplulation densitty, and C=nutient Concentration in growth chamber (see Table 4.1 of Edelstein-Keshet, p. 122)
with paramerts a1, a2, Equations (19a_, (19b) in Edelestein-Keshet p. 127 (section 4.5, where they are called alpha1, alpha2). a1 and a2 can be symbolic or numeric. Try:
ChemoStat(N,C,a1,a2);
ChemoStat(N,C,2,3); **(2)**

>

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[37, 33]$$

$$sols := \left[[N = 0, C = 60], \left[N = \frac{84322}{37}, C = \frac{1}{37}\right]\right] \quad (3)$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[99, 18]$$

$$sols := \left[[N = 0, C = 60], \left[N = \frac{84322}{37}, C = \frac{1}{37}\right]\right] \quad (4)$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[64, 24]$$

$$sols := \left[[N = 0, C = 60], \left[N = \frac{84322}{37}, C = \frac{1}{37}\right]\right] \quad (5)$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[78, 45]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(6)}$$

> $a1 := rand(1..100)(\,) : a2 := rand(1..100)(\,) : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[66, 24]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(7)}$$

> $a1 := rand(1..100)(\,) : a2 := rand(1..100)(\,) : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[59, 89]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(8)}$$

> $a1 := rand(1..100)(\,) : a2 := rand(1..100)(\,) : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[95, 97]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(9)}$$

> $a1 := rand(1..100)(\,) : a2 := rand(1..100)(\,) : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[60, 66]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(10)}$$

> $a1 := rand(1..100)(\,) : a2 := rand(1..100)(\,) : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[79, 47]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(11)}$$

> $a1 := rand(1..100)(\,) : a2 := rand(1..100)(\,) : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[32, 10]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(12)}$$

> $a1 := rand(1..100)(\,) : a2 := rand(1..100)(\,) : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[9, 47]$$

(13)

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(13)}$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[100, 9]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(14)}$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[48, 56]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(15)}$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[39, 28]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(16)}$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[16, 40]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(17)}$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[94, 17]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(18)}$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[100, 56]$$

$$sols := \left[[N=0, C=60], \left[N=\frac{84322}{37}, C=\frac{1}{37}\right] \right] \quad \textbf{(19)}$$

> $a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);$

$$[77, 49]$$

(20)

$$sols := \left[[N=0, C=60], \left[N = \frac{84322}{37}, C = \frac{1}{37} \right] \right] \quad \textbf{(20)}$$

```
> a1 := rand(1..100)() : a2 := rand(1..100)() : [a1, a2]; sols := SEquP(ChemoStat(N, C, a1, a2), [N, C]);
```

$$[54, 35]$$

$$sols := \left[[N=0, C=60], \left[N = \frac{84322}{37}, C = \frac{1}{37} \right] \right] \quad \textbf{(21)}$$

> *#For all 20 trials, the same two equilibrium points appeared*$\cdot$ $(N=0, C=60)$ **and** $\left(N = \frac{84322}{37}, C = \frac{1}{37}\right)$

#The non−trivial equilibrium was stable in each case, while the trivial one was unstable. Therefore, a stable equilibrium was obtained 20 out of 20 times.

Problem 3

read`*DMB.txt*`

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help (ProcedureName); for example Help(Feig);

For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(1)**

SIRSdemo(1000, 400, 1, 1, 0.01, 10);

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 0.01, *and letting it run until time t=*, 10

with population size, 1000, *and fixed parameters nu=*, 1, *and gamma=*, 1

*where we change beta from 0.2*nu/N to 4*nu/N*

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is, $\frac{1}{1000}$

We start with , 400, *infected individuals, 0 removed and hence*, 600, *susceptible*

We will show what happens once time is close to, 10

beta is , $\frac{1}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [999.6693512, 0.04464970605]], [9.99, [999.6721666, 0.04424784393]], [10.00, [999.6749582, 0.04384959883]], [10.01, [999.6777263, 0.04345493819]]]

beta is , $\frac{3}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [998.8686058, 0.2679278925]], [9.99, [998.8764375, 0.2660514879]], [10.00, [998.8842153, 0.2641882307]], [10.01, [998.8919395, 0.2623380288]]]

beta is , $\frac{1}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [995.5661036, 1.464972088]], [9.99, [995.5885005, 1.457614750]], [10.00, [995.6107835, 1.450294524]], [10.01, [995.6329532, 1.443011223]]]

beta is , $\frac{7}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [982.9907292, 6.871557578]], [9.99, [983.0448236, 6.850124744]], [10.00, [983.0987363, 6.828761355]], [10.01, [983.1524679, 6.807467168]]]

beta is , $\frac{9}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [944.9550913, 25.07830676]], [9.99, [945.0414764, 25.04080455]], [10.00, [945.1276722, 25.00337789]], [10.01, [945.2136792, 24.96602657]]]

beta is , $\frac{11}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [866.8575732, 64.57449614]], [9.99, [866.9275067, 64.54449698]], [10.00, [866.9972772, 64.51456141]], [10.01, [867.0668854, 64.48468924]]]

beta is , $\frac{13}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [764.2055840, 117.1693099]], [9.99, [764.2277964, 117.1616555]], [10.00, [764.2499053, 117.1540354]], [10.01, [764.2719113, 117.1464495]]]

beta is , $\frac{3}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [667.4467215, 166.2827762]], [9.99, [667.4446531, 166.2847218]], [10.00, [667.4425717, 166.2866623]], [10.01, [667.4404774, 166.2885977]]]

beta is , $\frac{17}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [588.7326192, 205.7837146]], [9.99, [588.7278789, 205.7854544]], [10.00, [588.7231678, 205.7871777]], [10.01, [588.7184858, 205.7888844]]]

beta is , $\frac{19}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [526.3391708, 236.9273141]], [9.99, [526.3371276, 236.9274194]], [10.00, [526.3351118, 236.9275155]], [10.01, [526.3331234, 236.9276024]]]

beta is , $\frac{21}{10}$, *times the threshold value* *the long-term behavior is*

[[9.98, [476.0755911, 261.9954214]], [9.99, [476.0755589, 261.9947893]], [10.00, [476.0755398, 261.9941570]], [10.01, [476.0755336, 261.9935246]]]

beta is , $\frac{23}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [434.6820669, 282.6567829]], [9.99, [434.6827642, 282.6561293]], [10.00, [434.6834631, 282.6554802]], [10.01, [434.6841635, 282.6548356]]]

beta is , $\frac{5}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [399.9447066, 300.0130619]], [9.99, [399.9454130, 300.0126472]], [10.00, [399.9461153, 300.0122378]], [10.01, [399.9468135, 300.0118336]]]

beta is , $\frac{27}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [370.3498315, 314.8101124]], [9.99, [370.3503056, 314.8099378]], [10.00, [370.3507743, 314.8097672]], [10.01, [370.3512378, 314.8096006]]]

beta is , $\frac{29}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [344.8255549, 327.5761726]], [9.99, [344.8257953, 327.5761533]], [10.00, [344.8260313, 327.5761363]], [10.01, [344.8262630, 327.5761215]]]

beta is , $\frac{31}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [322.5856476, 338.7004341]], [9.99, [322.5857299, 338.7004867]], [10.00, [322.5858094, 338.7005401]], [10.01, [322.5858863, 338.7005943]]]

beta is , $\frac{33}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [303.0363872, 348.4783244]], [9.99, [303.0363869, 348.4783944]], [10.00, [303.0363852, 348.4784644]], [10.01, [303.0363821, 348.4785344]]] *beta is* , $\frac{7}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [285.7191728, 357.1389626]], [9.99, [285.7191408, 357.1390237]], [10.00, [285.7191083, 357.1390844]], [10.01, [285.7190753, 357.1391447]]]

beta is , $\frac{37}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [270.2735369, 364.8628645]], [9.99, [270.2735002, 364.8629086]], [10.00, [270.2734634, 364.8629522]], [10.01, [270.2734267, 364.8629953]]]

beta is , $\frac{39}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [256.4121907, 371.7940209]], [9.99, [256.4121603, 371.7940490]], [10.00, [256.4121301, 371.7940766]], [10.01, [256.4121001, 371.7941037]]] **(2)**

Problem 4

read `DMB.txt`

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help (ProcedureName); for example Help(Feig);

For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(1)**

read `L18.txt`

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help (ProcedureName); for example Help(Feig);

For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(2)**

$HWgE(100, 1000);$

$$0.5500000000 \quad (3)$$

$HWgE(100, 1000);$

$$0.5280000000 \quad (4)$$

$HWgE(100, 1000);$

$$0.5710000000 \quad (5)$$

$HWgE(100, 1000);$

$$0.5480000000 \quad (6)$$

$HWgE(100, 1000);$

$$0.5510000000 \quad (7)$$

$HWgE(100, 1000);$

$$0.5430000000 \quad (8)$$

$HWgE(100, 1000);$

$$0.5510000000 \quad (9)$$

$HWgE(100, 1000);$

$$0.5410000000 \quad (10)$$

$HWgE(100, 1000);$

$$0.5690000000 \quad (11)$$

$HWgE(100, 1000);$

$$0.5380000000 \quad (12)$$

#the average is about 0.549 and the range is 0.043 (max-min), thus the values are really close. This would be the probability that only one genotype survuves is roughly the mean since the function estimates which one survives. There is about a 55% chance that a random preference matrix will lead to only one genotype surviving in the long run.