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Homework #17

1.) $dx/dt = -(x-1)(x-4)(x-7)(x-8) = f(x(t))$

eq. pt.s : $x = 1, 4, 7, 8$

$f'(x(t)) = -(x-4)(x-7)(x-8) - (x-1)(x-7)(x-8)$
$- (x-1)(x-4)(x-8) - (x-1)(x-4)(x-7)$

$f'(1) = -(-3)(-6)(-7) > 0$
$\Rightarrow x = 1$ is stable

$f'(4) = -(3)(-3)(-4) < 0$
$\Rightarrow 4$ is unstable

$f'(7) = -(6)(3)(-1) > 0$
$\Rightarrow 7$ is stable

$f'(8) = -(7)(4)(1) < 0$
$\Rightarrow 8$ is unstable

2.)

$f_1 = dx/dt = 1 - 3x/1+y+z = 0 \Rightarrow x = 1/3(1+y+z)$

$f_2 = dy/dt = 1 - 3y/1+x+z = 0 \Rightarrow y = 1/3(1+x+z)$

$f_3 = dz/dt = 1 - 3z/1+x+y = 0 \Rightarrow z = 1/3(1+x+y)$

$y = 1/3(1 + 1/3 + y/3 + z/3 + z) = 4/9 + y/9 + 4/9\, z$

$\Rightarrow y = 1/2 + 1/2 z \Rightarrow 1/2 = y - 1/2 z$

$z = 1/3(1 + 1/3 + y/3 + z/3 + y) = 4/9 + z/9 + 4/9\, y$

$z = 1/2 + 1/2 y \Rightarrow 1/2 = z - 1/2 y$

$\Rightarrow$ SSS: $x = 1,\ y = 1,\ z = 1$

$$Jac = \begin{bmatrix} \partial f_1/\partial x & \partial f_1/\partial y & \partial f_1/\partial z \\ \partial f_2/\partial x & \partial f_2/\partial y & \partial f_2/\partial z \\ \partial f_3/\partial x & \partial f_3/\partial y & \partial f_3/\partial z \end{bmatrix} = \begin{bmatrix} -3/1+y+z & 3x/(1+y+z)^2 & 3x/(1+y+z)^2 \\ 3y/(1+x+z)^2 & -3/1+x+z & 3y/(1+x+z)^2 \\ 3z/(1+x+y)^2 & 3z/(1+x+y)^2 & -3/1+x+y \end{bmatrix}$$

$$J(1,1,1) = \begin{bmatrix} -1 & 1/3 & 1/3 \\ 1/3 & -1 & 1/3 \\ 1/3 & 1/3 & -1 \end{bmatrix}$$

$\lambda = -1/3, -4/3, -4/3$

$\Rightarrow$ point $(1,1,1)$ is a stable steady-state

$$|J - \lambda I| = \begin{vmatrix} -1-\lambda & 1/3 & 1/3 \\ 1/3 & -1-\lambda & 1/3 \\ 1/3 & 1/3 & -1-\lambda \end{vmatrix}$$

3.)

$f_1 = dx/dt = 1 - x/(1+y+z) = 0 \Rightarrow x = 1+y+z$

$f_2 = dy/dt = 1 - y/(1+x+z) = 0 \Rightarrow y = 1+x+z$

$f_3 = dz/dt = 1 - z/(1+y+x) = 0 \Rightarrow z = 1+y+x$

$y = 1+1+y+z+z$

$z = -1, y = -1, x = -1$

$z = 1+y+1+y+z$

$$J = \begin{bmatrix} \partial f_1/\partial x & \partial f_1/\partial y & \partial f_1/\partial z \\ \partial f_2/\partial x & \partial f_2/\partial y & \partial f_2/\partial z \\ \partial f_3/\partial x & \partial f_3/\partial y & \partial f_3/\partial z \end{bmatrix} = \begin{bmatrix} -1/(1+y+z) & x/(1+y+z)^2 & x/(1+y+z)^2 \\ y/(1+x+z)^2 & -1/(1+x+z) & y/(1+x+z)^2 \\ z/(1+y+z)^2 & z/(1+y+z)^2 & -1/(1+y+z) \end{bmatrix}$$

$$J(-1,-1,-1) = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \quad |J - \lambda I| = \begin{vmatrix} 1-\lambda & -1 & -1 \\ -1 & 1-\lambda & -1 \\ -1 & -1 & 1-\lambda \end{vmatrix}$$

$\lambda = -1, 2, 2$ $\Rightarrow$ $(-1,-1,-1)$ is a stable steady-state