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Dynamic Model in Bio - HW 17

① $\frac{dx}{dt} = -(x-1)(x-4)(x-7)(x-8)$

equilibrium points: $x=1, 4, 7, 8$

$x=1 \Rightarrow (1-4)(1-7)(1-8) = -126 \Rightarrow (-1)(-126) = 126 \Rightarrow \text{unstable}$

$x=4 \Rightarrow (4-1)(4-7)(4-8) = 36 \Rightarrow (-1)(36) = -36 \Rightarrow \text{stable}$

$x=7 \Rightarrow (7-1)(7-4)(7-8) = -18 \Rightarrow (-1)(-18) = 18 \Rightarrow \text{unstable}$

$x=8 \Rightarrow (8-1)(8-4)(8-7) = 28 \Rightarrow (-1)(28) = -28 \Rightarrow \text{stable}$

② $\frac{dx}{dt} = 1 - \frac{3x}{1+y+z}$ $\frac{dy}{dt} = 1 - \frac{3y}{1+x+z}$ $\frac{dz}{dt} = 1 - \frac{3z}{1+x+y}$

$$\begin{cases} 3x - y - z = 1 \\ -x + 3y - z = 1 \\ -x - y + 3z = 1 \end{cases} \Rightarrow (x, y, z) = (1, 1, 1)$$

$$J = \begin{pmatrix} \frac{-3}{1+y+z} & \frac{3x}{(1+y+z)^2} & \frac{3x}{(1+y+z)^2} \\ \frac{3y}{(1+x+z)^2} & \frac{-3}{1+x+z} & \frac{3y}{(1+x+z)^2} \\ \frac{3z}{(1+x+y)^2} & \frac{3z}{(1+x+y)^2} & \frac{-3}{1+x+y} \end{pmatrix}$$

$$J(1, 1, 1) = \begin{pmatrix} -1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & -1 \end{pmatrix}$$

$$\lambda_1 = -\frac{1}{3} \quad \lambda_2 = -\frac{4}{3} \quad \lambda_3 = -\frac{4}{3}$$

$\Rightarrow$ All negative so $(1, 1, 1)$ is asymptotically stable

$$\textcircled{3} \quad \frac{dx}{dt} = 1 - \frac{x}{1+y+z} \quad \frac{dy}{dt} = 1 - \frac{y}{1+x+z} \quad \frac{dz}{dt} = 1 - \frac{z}{1+x+y}$$

$$x = 1+y+z$$

$$y = 1+x+z$$

$$z = 1+x+y$$

$$(x, y, z) = (-1, -1, -1)$$

$$J(x, y, z) = \begin{pmatrix} \frac{-1}{1+y+z} & \frac{x}{(1+y+z)^2} & \frac{x}{(1+y+z)^2} \\ \frac{y}{(1+x+z)^2} & \frac{-1}{1+x+z} & \frac{y}{(1+x+z)^2} \\ \frac{z}{(1+x+y)^2} & \frac{z}{(1+x+y)^2} & \frac{-1}{1+x+y} \end{pmatrix}$$

$$J(-1, -1, -1) = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$\lambda = -1 \quad \lambda = 2 \quad \lambda = 2$$

unstable

$$\textcircled{4} \quad a_1 = 2 \quad a_2 = 5$$