HW 17

① equ. points $\{1, 4, 7, 8\}$

$F'(x) = -(x-1)(x-4)(x-7) - (x-1)(x-4)(x-8) - (x-1)(x-7)(x-8) - (x-4)(x-7)(x-8)$

$F'(1) = -(-3)(-6)(-7) = 126$ UNS. $\quad F'(4) = -36$ STABLE

$F'(7) = 18$ UNST. $\quad F'(8) = -28$ STABLE

Stable Steady States $\boxed{\{4, 8\}}$

② $\left.\begin{aligned} 1+y+z &= 3x \\ 1+x+z &= 3y \\ 1+x+y &= 3z \end{aligned}\right\}$ $(1,1,1)$

$$J(x,y,z) = \begin{bmatrix} \frac{-3}{1+y+z} & \frac{3x}{(1+y+z)^2} & \frac{3x}{(1+y+z)^2} \\ \frac{3y}{(1+x+z)^2} & \frac{-3}{1+x+z} & \frac{3y}{(1+x+z)^2} \\ \frac{3z}{(1+y+x)^2} & \frac{3z}{(1+z+y)^2} & \frac{-3}{1+x+y} \end{bmatrix} \quad J(1,1,1) = \begin{bmatrix} -1 & 1/3 & 1/3 \\ 1/3 & -1 & 1/3 \\ 1/3 & 1/3 & -1 \end{bmatrix}$$

$\lambda = -1/3, -1/3, -1/3 < 0 \quad \therefore$ STABLE

③ $y = z = x = -1$

$$J(-1,-1,-1) = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$\lambda = -1 < 0$

$\left.\begin{aligned} 2 &> 0 \\ 2 &> 0 \end{aligned}\right\} \therefore$ UNSTABLE

④ $(1,0,0)$ UNSTABLE

$(2/3, 1/3, 0)$ UNSTABLE