Homework for Lecture 17 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Nov. 3, 2025.

Subject: hw17

with an attachment hw17FirstLast.pdf

1. Find all the equilibrium points and stable equilibrium points of the following one-dimensional dynmaical system

$$\frac{dx}{dt} = -(x-1)(x-4)(x-7)(x-8)$$

$x=1: F'(1) = -[(1-4)(1-7)(1-8)] = 126 \to$ unstable
$x=4: F'(4) = -[(4-1)(4-7)(4-8)] = -36 \to$ stable
$x=7: F'(7) = -[(7-1)(7-4)(7-8)] = 18 \to$ unstable
$x=8: F'(8) = -[(8-1)(8-4)(7-8)] = -28 \to$ stable
$x=4$ & $x=8$ are stable

2. (You can use Maple for the eigenvalues, but not for the Jacobian)

Find all stable equilibria of the 3-dimensinal dynamical system:

$$\frac{dx}{dt} = 1 - \frac{3x}{1+y+z}$$
$$\frac{dy}{dt} = 1 - \frac{3y}{1+x+z}$$
$$\frac{dz}{dt} = 1 - \frac{3z}{1+x+y}$$

$\to x = \frac{1+y+z}{3} \to \frac{\partial F_1}{\partial x} = \frac{-3}{1+y+z}$
$x - y = \frac{y-x}{3} \to x = y$
$\to y = \frac{1+x+z}{3} \to \frac{\partial F_1}{\partial y} = \frac{3x}{(1+y+z)^2}$
$y = z$
$z = \frac{1+x+y}{3} \to \frac{\partial F_1}{\partial z} \to \frac{3x}{(1+y+z)^2}$
$s = \frac{1+2s}{3} \to s = 1$ $(1,1,1)$

$$J = \begin{pmatrix} -1 & 1/3 & 1/3 \\ 1/3 & -1 & 1/3 \\ 1/3 & 1/3 & -1 \end{pmatrix}$$

$\lambda_1 = -1/3$ $\lambda_2 = -4/3$ $\lambda_3 = -4/3$
Stable

3. Find all the the equilibria and stable equilibria of the 3-dimensinal dynamical system:

$z = y = x$ $\quad s = 1+2s \to s = -1$ $\quad (-1,-1,-1)$

$$\frac{dx}{dt} = 1 - \frac{x}{1+y+z}$$
$$\frac{dy}{dt} = 1 - \frac{y}{1+x+z}$$
$$\frac{dz}{dt} = 1 - \frac{z}{1+x+y}$$

$\frac{\partial F_1}{\partial x} = -\frac{1}{1+y+z}$
$\frac{\partial F_1}{\partial y} = \frac{x}{(1+y+z)^2}$
$\frac{\partial F_1}{\partial z} = \frac{x}{(1+y+z)^2}$

$$J = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

$\lambda_1 = -1$ $\lambda_2 = 2$ $\lambda_3 = 2$

4. What are the equilibria, and stable equilibria of the Chemostat model with parameters $a_1 = 2$ and $a_2 = 5$?

$$N = a_1 \frac{CN}{C+1} - N = f(N,C)$$
$$C = -\frac{CN}{C+1} - C + a_2 = g(N,C)$$

$$\left(\frac{a_1(a_2 a_1 - a_2 - 1)}{a_1 - 1}, \frac{1}{a_1 - 1} \right)$$

$f_N = a_1 \frac{C}{C+1} - 1$ $\quad f_C = a_1 \frac{N}{(C+1)^2}$
$g_N = -\frac{C}{C+1}$ $\quad g_C = -\frac{N}{(C+1)^2} - 1$

$E_0 = (0,5)$
$E_1 = \left(2(2a_2 - a_2 - 1), 1\right) = (8,1)$

$J(E_0) = \begin{pmatrix} 2/3 & 0 \\ -5/6 & -1 \end{pmatrix}$
unstable

$J(E_1) = \begin{pmatrix} 0 & 4 \\ -1/2 & -3 \end{pmatrix}$
$\lambda^2 + 3\lambda + 2$
$(\lambda+1)(\lambda+2)$
$-1 \quad -2$
Stable