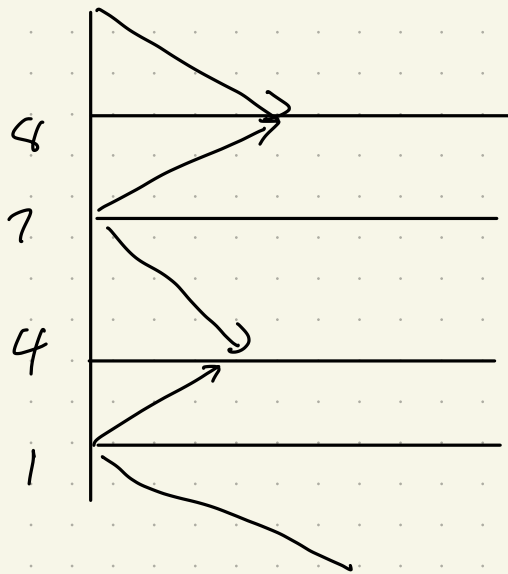


①  $x = 1, 4, 7, 8$

Stable :  $x = 4, 8$



② equilibrium =  $(1, 1, 1)$

$$\begin{bmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} & \frac{df_1}{dz} \\ \frac{df_2}{dx} & \frac{df_2}{dy} & \frac{df_2}{dz} \\ \frac{df_3}{dx} & \frac{df_3}{dy} & \frac{df_3}{dz} \end{bmatrix} = \begin{bmatrix} \frac{-3}{1+y+z} & \frac{3x}{(1+y+z)^2} & \frac{3x}{(1+y+z)^2} \\ \frac{3y}{(1+x+z)^2} & \frac{-3}{1+x+z} & \frac{3y}{(1+x+z)^2} \\ \frac{3z}{(1+x+y)^2} & \frac{3z}{(1+x+y)^2} & \frac{-3}{1+x+y} \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1/3 & 1/3 \\ 1/3 & -1 & 1/3 \\ 1/3 & 1/3 & -1 \end{bmatrix} \quad \lambda = -1/3, -4/3, -4/3$$

③ equilibrium =  $(-1, -1, -1)$

$$\begin{bmatrix} \frac{-1}{1+y+z} & \frac{x}{(1+y+z)^2} & \frac{x}{(1+y+z)^2} \\ \frac{y}{(1+x+z)^2} & \frac{-1}{1+x+z} & \frac{y}{(1+x+z)^2} \\ \frac{z}{(1+x+y)^2} & \frac{z}{(1+x+y)^2} & \frac{-1}{1+x+y} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$\lambda = 3, 0, 0$

④  $N \left( \frac{a_1 \cdot c}{1+c} - 1 \right) = \frac{dN}{dt}$

$$\frac{dc}{dt} = \frac{-c}{1+c} N - c + a_2$$

①  $a_1 = 2$   
②  $a_2 = 5$

$$-\frac{c}{1+c} N - c + 5$$

Solve for  $\frac{dN}{dt} = 0$   $\frac{dC}{dt} = 0$

$\frac{2C}{1+C} = 1$   $C=1$  inner

$N=0$   $\frac{-C}{1+C}(0) - (1+5) = 0$   
 $C=5$

$C=1$

$\frac{-1}{1+1}(N) - (1+5) = 0$

$N=8$

$(8, 1)$

$$J = \begin{bmatrix} \frac{2C}{1+C} - 1 & \frac{2N}{(1+C)^2} \\ \frac{-C}{1+C} & \frac{-N}{(1+C)^2} - 1 \end{bmatrix}$$

①  $(0, 5)$

$$\begin{bmatrix} 2/3 & 0 \\ -5/6 & -1 \end{bmatrix}$$

$\lambda = 2/3, -1$

unstable

②  $(8, 1)$

$$\begin{bmatrix} 0 & 4 \\ -1/2 & 3 \end{bmatrix}$$

$\lambda = -1, -2$

stable