Homework for Lecture 17 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Nov. 3, 2025.

Subject: hw17

with an attachment hw17FirstLast.pdf

1. Find all the equilibrium points and stable equilibrium points of the following one-dimensional dynmaical system

$$\frac{dx}{dt} = -(x-1)(x-4)(x-7)(x-8) \quad .$$

2. (You can use Maple for the eigenvalues, but not for the Jacobian)

Find all stable equilibria of the 3-dimensinal dynamical system:

$$\frac{dx}{dt} = 1 - \frac{3x}{1+y+z} \quad ,$$
$$\frac{dy}{dt} = 1 - \frac{3y}{1+x+z} \quad ,$$
$$\frac{dz}{dt} = 1 - \frac{3z}{1+x+y} \quad .$$

3. Find all the the equilibria and stable equilibria of the 3-dimensinal dynamical system:

$$\frac{dx}{dt} = 1 - \frac{x}{1+y+z} \quad ,$$
$$\frac{dy}{dt} = 1 - \frac{y}{1+x+z} \quad ,$$
$$\frac{dz}{dt} = 1 - \frac{z}{1+x+y} \quad .$$

4. What are the equilibria, and stable equilibria of the Chemostat model with parameters $a_1 = 2$ and $a_2 = 5$?

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1. Find all the equilibrium points and stable equilibrium points of the following one-dimensional dynmaical system

$$\frac{dx}{dt} = -(x-1)(x-4)(x-7)(x-8) \quad .$$

$0 = -(x-1)(x-4)(x-7)(x-8)$

$x = 1, 4, 7, 8$

$f'(x) = -(x-4)(x-7)(x-8) - (x-1)(x-7)(x-8) - (x-1)(x-4)(x-8) - (x-1)(x-4)(x-7)$

$f'(1) = (-)(-)(-)(-) = +$

$f'(4) = 0 + (-)(+)(-)(-) = -$

$f'(7) = 0 - 0 + (-)(+)(+)(-) = +$

$f'(8) = 0 - 0 - 0 + (-)(+)(+)(+) = -$

Since $f'(4)$ and $f'(8) < 0$, $\{4, 8\}$ are Stable equilibria

2. (You can use Maple for the eigenvalues, but not for the Jacobian)

Find all stable equilibria of the 3-dimensinal dynamical system:

$$\frac{dx}{dt} = 1 - \frac{3x}{1+y+z} \quad ,$$

$$\frac{dy}{dt} = 1 - \frac{3y}{1+x+z} \quad ,$$

$$\frac{dz}{dt} = 1 - \frac{3z}{1+x+y} \quad .$$

$3x(1+y+z)^{-1}$

$0 + 3x(-1)(1)(1+y+z)^{-2}$

$0 = 1 - \frac{3x}{1+y+z}$

$1 = \frac{3x}{1+y+z}$

$1+y+z = 3x$

$y = 3x - z - 1$

$y = x$

$y = 1$

$0 = 1 - \frac{3y}{1+x+z}$

$1 = \frac{9x-3z-3}{1+x+z}$

$1+x+z = 9x - 3z - 3$

$4z = 8x - 4$

$z = 2x - 1$

$z = 1$

$0 = 1 - \frac{3z}{1+x+y}$

$1 = \frac{6x-3}{1+x+x}$

$1 = \frac{6x-3}{1+2x}$

$1 + 2x = 6x - 3$

$x = 1$

$$J = \begin{bmatrix} \frac{-3}{1+y+z} & \frac{3x}{(1+y+z)^2} & \frac{3x}{(1+y+z)^2} \\ \frac{3y}{(1+x+z)^2} & \frac{-3}{(1+x+z)} & \frac{3y}{(1+x+z)^2} \\ \frac{3z}{(1+x+y)^2} & \frac{3z}{(1+x+y)^2} & \frac{-3}{(1+y+y)} \end{bmatrix}$$

$$J(1,1,1) = \begin{bmatrix} -1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & -1 \end{bmatrix}$$

Found eigenvalues w/ maple: $\lambda = -\frac{1}{3}, -\frac{4}{3}, -\frac{4}{3}$

Since $\lambda < 0$, Stable eq.

3. Find all the the equilibria and stable equilibria of the 3-dimensinal dynamical system:

$$\frac{dx}{dt} = 1 - \frac{x}{1+y+z} \quad ,$$

$$\frac{dy}{dt} = 1 - \frac{y}{1+x+z} \quad ,$$

$$\frac{dz}{dt} = 1 - \frac{z}{1+x+y} \quad .$$

$0 = 1 - \frac{x}{1+y+z}$
$1+y+z = x$
$y = x - z - 1$
$y = x$

$y = -1$

$0 = 1 - \frac{y}{1+x+z}$
$1 = \frac{x-z-1}{1+x+z}$
$1+x+z = x-z-1$
$2z = -2$
$z = -1$

$0 = 1 - \frac{z}{1+x+y}$
$1 = \frac{-1}{1+x+y}$
$1+x+x = -1$
$2x = -2$
$x = -1$

$$J = \begin{bmatrix} \frac{-1}{(1+y+z)} & \frac{x}{(1+y+z)^2} & \frac{x}{(1+y+z)^2} \\ \frac{y}{(1+x+z)^2} & \frac{-1}{(1+x+z)} & \frac{y}{(1+x+z)^2} \\ \frac{z}{(1+x+y)^2} & \frac{z}{(1+x+y)^2} & \frac{-1}{(1+x+y)} \end{bmatrix} \qquad J(-1,-1,-1) = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$\lambda = -1, 2, 2$

$\lambda > 0 \therefore$ unstable equilibrium

4. What are the equilibria, and stable equilibria of the Chemostat model with parameters $a_1 = 2$ and $a_2 = 5$?

$\frac{dN}{dt} = a_1 \frac{CN}{C+1} - N \quad = 2\frac{CN}{C+1} - N$

$\frac{dC}{dt} = -\frac{CN}{C+1} + a_2 \quad = -\frac{CN}{C+1} + 5$

$0 = 2\frac{CN}{C+1} - N$
$N = 2\frac{CN}{C+1}$
$CN + N = 2CN$
$C + 1 = 2C$
$1 = C$

$(10, 1)$

$0 = -\frac{CN}{C+1} + 5$
$5 = \frac{CN}{C+1}$
$5C + 5 = CN$
$5 + \frac{5}{C} = N$

$10 = N$

$$J = \begin{bmatrix} \frac{C-1}{C+1} & \frac{2N}{(C+1)^2} \\ \frac{-C}{C+1} & \frac{N}{(C+1)^2} - 1 \end{bmatrix} \qquad J(10,1) = \begin{bmatrix} 0 & 5 \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

$(0-\lambda)(\frac{3}{2}-\lambda) + \frac{5}{2}$
$-\frac{3}{2}\lambda + \lambda^2 + \frac{5}{2}$
$\lambda^2 - \frac{3}{2}\lambda + \frac{5}{2} = 0$

$$\frac{\frac{3}{2} \pm \sqrt{\frac{9}{4} - 10}}{2} = \frac{\frac{3}{2} \pm \sqrt{\frac{-31}{4}}}{2}$$

Since the real part of λ is > 0
the pt $(10, 1)$ is not steady