

Homework for Lecture 16 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Nov. 3, 2025.

Subject: hw16

with an attachment hw16FirstLast.pdf

1. For the generalized Hardy-Weinberg model given by $HWg(u,v,M)$ where M is a 3 by 3 preference matrix

Type

```
M:=RandMat(3,30); T:=HWg(u,v,M); SSSgN(T,[u,v]);
```

40 times and record in how many cases only one of the alleles AA, Aa, aa will survive (i.e. the output is close to [0,1], [1,0], or [0,0]) and in how many of them they will also survive.

2. for each of $b=c=0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4$

Using

```
SSSgN(AllenSIR(a,b,c,x,y),[x,y]);
```

for $a=(b+c)/2$, $a=(b+c)$, $a=1.5*(b+c)$, $a=2*(b+c)$, $a=10*(b+c)$, $a=100*(b+c)$

Confirm, in each case, that in the Linda Allen model, for $a = (b + c)/2$ in the long run there are no infected individuals, but after $a = (b + c)$, they will start showing up. Also confirm that even for large a , they will not all be infected but x_n and y_n tend to some number.

1. For the generalized Hardy-Weinberg model given by $HWg(u,v,M)$ where M is a 3 by 3 preference matrix

Type

$M := \text{RandMat}(3,30); T := HWg(u,v,M); SSSgN(T, [u,v]);$

40 times and record in how many cases **only one of the alleles AA, Aa, aa will survive** (i.e. the output is close to [0,1], [1,0], or [0,0]) and in how many of them they will also survive.

22 cases had only 1 allele survive

> read 'DMB.txt';		
	For a list of the Main procedures type: Help(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig);	(1)
> Help()	For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig);	(2)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	The available procedures are: AllenSIR, AllenSIRg, ChaosStory, Feig, HW, NicholsonBailey, NicholsonBaileyM, Orb, ORB, OrbK, RecToT, RecToTs, SS, SSg, SSr, SSS, SSSg, SSSgN, RR, RT	(3)
	$M := [[29, 23, 15], [26, 13, 6], [2, 27, 12]]$ $T := \left[\frac{116u^2 + 98uv + 13v^2}{4(24u^2 + 23uv - 8v^2 - 7u + 9v + 12)}, -\frac{34u^2 + 18uv + 20v^2 - 34u - 33v}{2(24u^2 + 23uv - 8v^2 - 7u + 9v + 12)} \right]$ $[0.9999999992, 7.999999985 \times 10^{-10}]$	(4)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[14, 7, 28], [7, 3, 14], [29, 15, 15]]$ $T := \left[-\frac{56u^2 + 28uv + 3v^2}{4(28u^2 + 42uv + 11v^2 - 27u + v - 15)}, \frac{114u^2 + 129uv + 26v^2 - 114u - 29v}{2(28u^2 + 42uv + 11v^2 - 27u + v - 15)} \right]$ $[5.085632845 \times 10^{-34}, 9.749093655 \times 10^{-17}]$	(5)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[8, 9, 17], [26, 18, 11], [20, 23, 13]]$ $T := \left[-\frac{16u^2 + 35uv + 9v^2}{2(16u^2 + 10uv + 3v^2 - 11u - 8v - 13)}, \frac{74u^2 + 73uv + 16v^2 - 74u - 34v}{2(16u^2 + 10uv + 3v^2 - 11u - 8v - 13)} \right]$ $[0.1986145182, 0.5214197860]$	(6)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[11, 5, 16], [11, 4, 27], [22, 8, 26]]$ $T := \left[-\frac{11u^2 + 8uv + v^2}{u^2 + 5uv + 5v^2 + 14u + 17v - 26}, \frac{76u^2 + 95uv + 31v^2 - 76u - 35v}{2(u^2 + 5uv + 5v^2 + 14u + 17v - 26)} \right]$ $[1.314787653 \times 10^{-341}, 1.244454961 \times 10^{-170}]$	(7)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[27, 7, 2], [12, 17, 5], [26, 3, 15]]$ $T := \left[\frac{108u^2 + 38uv + 17v^2}{4(14u^2 + 13uv + 24v^2 - 2u - 22v + 15)}, -\frac{56u^2 + 45uv - 9v^2 - 56u - 8v}{2(14u^2 + 13uv + 24v^2 - 2u - 22v + 15)} \right]$ $[1.000000000, -6.511136825 \times 10^{-817}]$	(8)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[10, 17, 28], [5, 25, 1], [30, 1, 19]]$ $T := \left[-\frac{40u^2 + 44uv + 25v^2}{4(29u^2 - 42v^2 - 20u + 36v - 19)}, \frac{116u^2 + 96uv - 23v^2 - 116u - 2v}{2(29u^2 - 42v^2 - 20u + 36v - 19)} \right]$ $[0.2956610970, 0.5451146075]$	(9)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[17, 28, 29], [20, 15, 24], [5, 17, 21]]$ $T := \left[\frac{68u^2 + 96uv + 15v^2}{4(4u^2 + 15uv - 5v^2 - 8u - v + 21)}, -\frac{68u^2 + 61uv + 26v^2 - 68u - 41v}{2(4u^2 + 15uv - 5v^2 - 8u - v + 21)} \right]$ $[0.5500928410, 0.3949476804]$	(10)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[25, 29, 2], [2, 12, 28], [19, 21, 4]]$ $T := \left[\frac{50u^2 + 31uv + 6v^2}{2(8u^2 - 31uv - 33v^2 + 13u + 41v + 4)}, -\frac{42u^2 + 60uv + 37v^2 - 42u - 49v}{2(8u^2 - 31uv - 33v^2 + 13u + 41v + 4)} \right]$ $[1.000000000, 7.914219430 \times 10^{-19}]$	(11)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[3, 8, 16], [9, 9, 14], [14, 21, 3]]$ $T := \left[-\frac{12u^2 + 34uv + 9v^2}{4(24u^2 + 42uv + 23v^2 - 24u - 29v - 3)}, \frac{60u^2 + 78uv + 26v^2 - 60u - 35v}{2(24u^2 + 42uv + 23v^2 - 24u - 29v - 3)} \right]$ $[0.08536612952, 0.5219484615]$	(12)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[28, 10, 17], [14, 12, 26], [23, 15, 7]]$ $T := \left[-\frac{28u^2 + 12uv + 3v^2}{5u^2 + 43uv + 22v^2 - 26u - 27v - 7}, \frac{80u^2 + 97uv + 29v^2 - 80u - 41v}{2(5u^2 + 43uv + 22v^2 - 26u - 27v - 7)} \right]$ $[1.000000000, -6.972498950 \times 10^{-141}]$	(13)
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);	$M := [[24, 14, 13], [22, 2, 24], [5, 27, 15]]$ $T := \left[\frac{48u^2 + 36uv + v^2}{2(21u^2 - 3uv - 34v^2 - 12u + 21v + 15)}, -\frac{36u^2 + 51uv + 49v^2 - 36u - 51v}{2(21u^2 - 3uv - 34v^2 - 12u + 21v + 15)} \right]$ $[0.003632274242, 0.3031559217]$	(13)

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[10, 25, 11], [12, 4, 12], [15, 30, 16]]$$

$$T := \left[\frac{20u^2 + 37uv + 2v^2}{2(uv - 22v^2 - 6u + 10v + 16)}, -\frac{52u^2 + 57uv + 38v^2 - 52u - 42v}{2(uv - 22v^2 - 6u + 10v + 16)} \right]$$

$$[0.003807647852, 0.2221871492]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[19, 15, 26], [10, 9, 30], [30, 27, 18]]$$

$$T := \left[-\frac{76u^2 + 50uv + 9v^2}{4(19u^2 + 52uv + 30v^2 - 20u - 21v - 18)}, \frac{112u^2 + 144uv + 48v^2 - 112u - 57v}{2(19u^2 + 52uv + 30v^2 - 20u - 21v - 18)} \right]$$

$$[0.01517208205, 0.3401869112]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[25, 2, 9], [5, 26, 23], [9, 18, 23]]$$

$$T := \left[\frac{50u^2 + 7uv + 13v^2}{2(30u^2 - 6uv + 8v^2 - 28u - 5v + 23)}, -\frac{36u^2 + 70uv + 15v^2 - 36u - 41v}{2(30u^2 - 6uv + 8v^2 - 28u - 5v + 23)} \right]$$

$$[1.000000000, 1.098901100 \times 10^{-10}]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[25, 11, 30], [28, 26, 24], [8, 2, 4]]$$

$$T := \left[-\frac{50u^2 + 39uv + 13v^2}{2(9u^2 + 17uv - 4v^2 - 30u - 18v - 4)}, \frac{76u^2 + 63uv - 76u - 26v}{2(9u^2 + 17uv - 4v^2 - 30u - 18v - 4)} \right]$$

$$[0.9999999980, 1.668209596 \times 10^{-9}]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[22, 21, 25], [29, 30, 1], [20, 11, 10]]$$

$$T := \left[-\frac{44u^2 + 50uv + 15v^2}{2(13u^2 - 13uv - 28v^2 - 25u + 8v - 10)}, \frac{45u^2 + 26uv - 9v^2 - 45u - 6v}{13u^2 - 13uv - 28v^2 - 25u + 8v - 10} \right]$$

$$[0.5807904580, 0.3704339174]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[19, 8, 8], [14, 27, 15], [7, 11, 5]]$$

$$T := \left[\frac{76u^2 + 44uv + 27v^2}{4(9u^2 - 9uv + 6v^2 + 5u + 16v + 5)}, -\frac{30u^2 + 34uv - v^2 - 30u - 26v}{2(9u^2 - 9uv + 6v^2 + 5u + 16v + 5)} \right]$$

$$[0.9999999992, 5.114246440 \times 10^{-10}]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[9, 21, 15], [16, 11, 25], [10, 29, 28]]$$

$$T := \left[\frac{36u^2 + 74uv + 11v^2}{4(12u^2 + 14uv - 15v^2 - 31u - 2v + 28)}, -\frac{50u^2 + 67uv + 43v^2 - 50u - 54v}{2(12u^2 + 14uv - 15v^2 - 31u - 2v + 28)} \right]$$

$$[1.256121508 \times 10^{-35}, 1.090521488 \times 10^{-17}]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[22, 8, 10], [4, 21, 19], [8, 11, 1]]$$

$$T := \left[\frac{88u^2 + 24uv + 21v^2}{4(5u^2 - 34uv - 8v^2 + 16u + 28v + 1)}, -\frac{3(12u^2 + 18uv + 3v^2 - 12u - 10v)}{2(5u^2 - 34uv - 8v^2 + 16u + 28v + 1)} \right]$$

$$[1.000000000, -4.054054054 \times 10^{-10}]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[1, 20, 28], [28, 26, 7], [8, 12, 28]]$$

$$T := \left[-\frac{2u^2 + 48uv + 13v^2}{2(7u^2 - 49uv - 35v^2 + 20u + 37v - 28)}, \frac{72u^2 + 43uv - 7v^2 - 72u - 19v}{2(7u^2 - 49uv - 35v^2 + 20u + 37v - 28)} \right]$$

$$[2.295494702 \times 10^{-939}, -3.373852241 \times 10^{-470}]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[7, 22, 22], [23, 2, 17], [5, 24, 16]]$$

$$T := \left[-\frac{14u^2 + 45uv + v^2}{2(4u^2 - 9uv + 23v^2 + 5u - 9v - 16)}, \frac{54u^2 + 50uv + 39v^2 - 54u - 41v}{2(4u^2 - 9uv + 23v^2 + 5u - 9v - 16)} \right]$$

$$[0.5427150870, 0.4361876580]$$

Page Break

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

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$$M := [[9, 18, 29], [2, 21, 12], [3, 17, 28]]$$

$$T := \left[\frac{36u^2 + 40uv + 21v^2}{4(5u^2 + 15uv + 20v^2 - 24u - 27v + 28)}, -\frac{64u^2 + 73uv + 8v^2 - 64u - 29v}{2(5u^2 + 15uv + 20v^2 - 24u - 27v + 28)} \right]$$

$$[7.398414725 \times 10^{-572}, 3.252959610 \times 10^{-286}]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[22, 7, 22], [17, 7, 5], [4, 30, 5]]$$
$$T := \left[-\frac{88u^2 + 48uv + 7v^2}{4(u^2 - 27uv - 23v^2 + 16u + 25v + 5)}, -\frac{52u^2 + 63uv + 28v^2 - 52u - 35v}{2(u^2 - 27uv - 23v^2 + 16u + 25v + 5)} \right]$$
$$\left[1.000000000, 4.708828564 \times 10^{-106} \right]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[3, 12, 27], [6, 29, 4], [27, 20, 13]]$$
$$T := \left[-\frac{12u^2 + 36uv + 29v^2}{4(38u^2 + 34uv - 18v^2 - 28u + 2v - 13)}, \frac{108u^2 + 114uv - 5v^2 - 108u - 24v}{2(38u^2 + 34uv - 18v^2 - 28u + 2v - 13)} \right]$$
$$[0.1733330917, 0.5428410970]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[21, 20, 19], [25, 7, 4], [5, 25, 3]]$$
$$T := \left[-\frac{84u^2 + 90uv + 7v^2}{4(2uv + 19v^2 - 18u - 23v - 3)}, \frac{48u^2 + 32uv + 22v^2 - 48u - 29v}{2(2uv + 19v^2 - 18u - 23v - 3)} \right]$$
$$[0.9189037440, 0.08052747750]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[6, 12, 2], [8, 6, 19], [16, 13, 5]]$$
$$T := \left[-\frac{12u^2 + 20uv + 3v^2}{2(7u^2 + 20uv + 21v^2 - 8u - 22v - 5)}, \frac{18u^2 + 24uv + 13v^2 - 18u - 16v}{7u^2 + 20uv + 21v^2 - 8u - 22v - 5} \right]$$
$$[0.06035227230, 0.4711966998]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[27, 14, 22], [6, 24, 30], [23, 30, 26]]$$
$$T := \left[\frac{27u^2 + 10uv + 6v^2}{8u^2 - 33uv - 10v^2 - 7u + 8v + 26}, -\frac{45u^2 + 65uv + 18v^2 - 45u - 30v}{8u^2 - 33uv - 10v^2 - 7u + 8v + 26} \right]$$
$$\left[1.000000000, -4.652850404 \times 10^{-111} \right]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

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$$M := [[13, 15, 16], [16, 6, 22], [22, 10, 6]]$$
$$T := \left[-\frac{26u^2 + 31uv + 3v^2}{2(19u^2 + 27uv + 20v^2 - 26u - 20v - 6)}, \frac{76u^2 + 77uv + 26v^2 - 76u - 32v}{2(19u^2 + 27uv + 20v^2 - 26u - 20v - 6)} \right]$$
$$[0.7548302220, 0.2365637790]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[7, 24, 25], [16, 4, 13], [29, 27, 3]]$$
$$T := \left[-\frac{7u^2 + 20uv + v^2}{44u^2 + 48uv + 33v^2 - 48u - 34v - 3}, \frac{2(27u^2 + 27uv + 9v^2 - 27u - 10v)}{44u^2 + 48uv + 33v^2 - 48u - 34v - 3} \right]$$
$$[0.3833021014, 0.5280305164]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[2, 11, 5], [21, 20, 8], [20, 6, 4]]$$
$$T := \left[-\frac{2u^2 + 16uv + 5v^2}{19u^2 - uv - 10v^2 - 17u - 6v - 4}, \frac{25u^2 + 16uv - 3v^2 - 25u - 7v}{19u^2 - uv - 10v^2 - 17u - 6v - 4} \right]$$
$$[0.3146199747, 0.5331476283]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[29, 9, 24], [18, 5, 26], [22, 12, 22]]$$
$$T := \left[\frac{116u^2 + 54uv + 5v^2}{4(5u^2 - 13uv - 11v^2 + 2u - 6v + 22)}, -\frac{92u^2 + 103uv + 33v^2 - 92u - 38v}{2(5u^2 - 13uv - 11v^2 + 2u - 6v + 22)} \right]$$
$$\left[1.865191270 \times 10^{-130}, 4.948219328 \times 10^{-65} \right]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[12, 12, 13], [24, 12, 11], [1, 17, 22]]$$
$$T := \left[\frac{3(4u^2 + 6uv + v^2)}{2(10u^2 + 19uv + 3v^2 - 15u - 8v + 11)}, -\frac{7u^2 + 5uv + 4v^2 - 7u - 7v}{10u^2 + 19uv + 3v^2 - 15u - 8v + 11} \right]$$
$$\left[2.515179309 \times 10^{-394}, -2.733005740 \times 10^{-197} \right]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[2, 12, 24], [24, 27, 13], [29, 22, 2]]$$
$$T := \left[-\frac{8u^2 + 72uv + 27v^2}{4(49u^2 + 48uv + 6v^2 - 49u - 31v - 2)}, \frac{106u^2 + 105uv + 8v^2 - 106u - 35v}{2(49u^2 + 48uv + 6v^2 - 49u - 31v - 2)} \right]$$
$$[0.2236131392, 0.5595034095]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[13, 12, 14], [1, 13, 7], [2, 20, 15]]$$
$$T := \left[\frac{13(4u^2 + 2uv + v^2)}{4(12u^2 + v^2 - 14u - 3v + 15)}, -\frac{32u^2 + 46uv + 14v^2 - 32u - 27v}{2(12u^2 + v^2 - 14u - 3v + 15)} \right]$$
$$\left[1.000000000, -1.064734846 \times 10^{-10} \right]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[29, 15, 18], [2, 13, 2], [5, 5, 29]]$$
$$T := \left[\frac{116u^2 + 34uv + 13v^2}{4(35u^2 + 45uv + 35v^2 - 35u - 51v + 29)}, -\frac{46u^2 + 36uv - 6v^2 - 46u - 7v}{2(35u^2 + 45uv + 35v^2 - 35u - 51v + 29)} \right]$$
$$\left[3.934369625 \times 10^{-1831}, 2.261335205 \times 10^{-916} \right]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[28, 9, 21], [8, 8, 15], [16, 9, 8]]$$
$$T := \left[-\frac{56u^2 + 17uv + 4v^2}{2(u^2 + 28uv + 8v^2 - 21u - 8v - 8)}, \frac{74u^2 + 81uv + 16v^2 - 74u - 24v}{2(u^2 + 28uv + 8v^2 - 21u - 8v - 8)} \right]$$
$$\left[1.000000000, 1.250000000 \times 10^{-10} \right]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[3, 26, 12], [10, 19, 7], [29, 23, 5]]$$
$$T := \left[-\frac{12u^2 + 72uv + 19v^2}{4(33u^2 + 25uv + 6v^2 - 31u - 20v - 5)}, \frac{82u^2 + 76uv + 11v^2 - 82u - 30v}{2(33u^2 + 25uv + 6v^2 - 31u - 20v - 5)} \right]$$
$$[0.2489565087, 0.5504940160]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[25, 7, 10], [17, 2, 20], [29, 24, 26]]$$
$$T := \left[\frac{50u^2 + 24uv + v^2}{2(12u^2 - 7uv - 16v^2 - 13u - 8v + 26)}, -\frac{39u^2 + 49uv + 21v^2 - 39u - 22v}{12u^2 - 7uv - 16v^2 - 13u - 8v + 26} \right]$$
$$\left[2.001942190 \times 10^{-148}, 8.633298773 \times 10^{-74} \right]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[27, 12, 22], [26, 6, 2], [20, 30, 5]]$$
$$T := \left[-\frac{54u^2 + 38uv + 3v^2}{2(10u^2 + 26uv + 21v^2 - 32u - 22v - 5)}, \frac{42u^2 + 39uv + 13v^2 - 42u - 16v}{10u^2 + 26uv + 21v^2 - 32u - 22v - 5} \right]$$
$$\left[1.000000000, 2.800973953 \times 10^{-10} \right]$$

> $M := \text{RandMat}(3, 30); T := \text{HWg}(u, v, M); \text{SSSgN}(T, [u, v]);$

$$M := [[17, 13, 28], [13, 26, 14], [24, 10, 15]]$$
$$T := \left[-\frac{34u^2 + 26uv + 13v^2}{2(20u^2 + 20uv - 17v^2 - 22u + 6v - 15)}, \frac{52u^2 + 51uv - v^2 - 52u - 12v}{20u^2 + 20uv - 17v^2 - 22u + 6v - 15} \right]$$
$$[0.2654568336, 0.5278668632]$$

2. for each of $b=c=0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4$

Using

`SSSgN(AllenSIR(a,b,c,x,y), [x,y]);`

for $a_1=(b+c)/2$, $a_2=(b+c)$, $a_3=1.5*(b+c)$, $a_4=2*(b+c)$, $a_5=10*(b+c)$, $a_6=100*(b+c)$

Confirm, in each case, that in the Linda Allen model, for $a = (b + c)/2$ in the long run there are no infected individuals, but after $a = (b + c)$, they will start showing up. Also confirm that even for large a , they will not all be infected but x_n and y_n tend to some number.

Q1

```
> AllenSIR(.1, .1, .1, x, y), [x, y]);
[0.8 x + y (1 - e-0.1 x), 0.1 - 0.1 y + y e-0.1 x]
> SSSgN(% [x, y]);
[7. × 10-10, 0.9999999986]
> AllenSIR(.15, .15, .15, x, y), [x, y]);
[0.70 x + y (1 - e-0.15 x), 0.15 - 0.15 y + y e-0.15 x]
> SSSgN(% [x, y]);
[4. × 10-10, 0.9999999991]
> AllenSIR(.2, .2, .2, x, y), [x, y]);
[0.6 x + y (1 - e-0.2 x), 0.2 - 0.2 y + y e-0.2 x]
> SSSgN(% [x, y]);
[3. × 10-10, 0.9999999993]
> AllenSIR(.25, .25, .25, x, y), [x, y]);
[0.50 x + y (1 - e-0.25 x), 0.25 - 0.25 y + y e-0.25 x]
> SSSgN(% [x, y]);
[0., 0.9999999998]
> AllenSIR(.25, .25, .25, x, y), [x, y]);
[0.50 x + y (1 - e-0.25 x), 0.25 - 0.25 y + y e-0.25 x]
> SSSgN(% [x, y]);
[0., 0.9999999998]
> AllenSIR(.3, .3, .3, x, y), [x, y]);
[0.4 x + y (1 - e-0.3 x), 0.3 - 0.3 y + y e-0.3 x]
> SSSgN(% [x, y]);
[2. × 10-10, 0.9999999996]
> AllenSIR(.35, .35, .35, x, y), [x, y]);
[0.30 x + y (1 - e-0.35 x), 0.35 - 0.35 y + y e-0.35 x]
> SSSgN(% [x, y]);
[2. × 10-10, 0.9999999996]
> AllenSIR(.4, .4, .4, x, y), [x, y]);
[0.2 x + y (1 - e-0.4 x), 0.4 - 0.4 y + y e-0.4 x]
> SSSgN(% [x, y]);
Error, (in sqfree) argument must be a polynomial or a rational function in lam [DMB.txt:746]
```

Q2

```
> AllenSIR(.2, .1, .1, x, y), [x, y]);
[0.8 x + y (1 - e-0.2 x), 0.1 - 0.1 y + y e-0.2 x] (3)
> SSSgN(% [x, y]);
[0.0023458826, 0.9952845412] (4)
> AllenSIR(.3, .15, .15, x, y), [x, y]);
[0.70 x + y (1 - e-0.3 x), 0.15 - 0.15 y + y e-0.3 x] (5)
> SSSgN(% [x, y]);
[0.0015293689, 0.9969310399] (6)
> AllenSIR(.4, .2, .2, x, y), [x, y]);
[0.6 x + y (1 - e-0.4 x), 0.2 - 0.2 y + y e-0.4 x] (7)
> SSSgN(% [x, y]);
[0.0011217441, 0.9977509080] (8)
> AllenSIR(.5, .25, .25, x, y), [x, y]);
[0.50 x + y (1 - e-0.5 x), 0.25 - 0.25 y + y e-0.5 x] (9)
> SSSgN(% [x, y]);
[0.0008777887, 0.9982409222] (10)
> AllenSIR(.6, .3, .3, x, y), [x, y]);
[0.4 x + y (1 - e-0.6 x), 0.3 - 0.3 y + y e-0.6 x] (11)
> SSSgN(% [x, y]);
[0.0007156888, 0.9985662474] (12)
> AllenSIR(.7, .35, .35, x, y), [x, y]);
[0.30 x + y (1 - e-0.7 x), 0.35 - 0.35 y + y e-0.7 x] (13)
> SSSgN(% [x, y]);
[0.0006012272, 0.9987958353] (14)
> AllenSIR(.8, .4, .4, x, y), [x, y]);
[0.2 x + y (1 - e-0.8 x), 0.4 - 0.4 y + y e-0.8 x] (15)
> SSSgN(% [x, y]);
Error, (in sqfree) argument must be a polynomial or a rational function in lam [DMB.txt:746]
```

values
start
showing
up

03

```

> AllenSIR( (.3, .1, .1, x, y), [x, y]);
[0.8 x + y (1 - e-0.3 x), 0.1 - 0.1 y + y e-0.3 x]
> SSSgN(%, [x, y]);
[0.1586702173, 0.6826595654]
> AllenSIR( (.45, .15, .15, x, y), [x, y]);
[0.70 x + y (1 - e-0.45 x), 0.15 - 0.15 y + y e-0.45 x]
> SSSgN(%, [x, y]);
[0.1549131987, 0.6901736030]
> AllenSIR( (.6, .2, .2, x, y), [x, y]);
[0.6 x + y (1 - e-0.6 x), 0.2 - 0.2 y + y e-0.6 x]
> SSSgN(%, [x, y]);
[0.1513070546, 0.6973858908]
>
> AllenSIR( (.75, .25, .25, x, y), [x, y]);
[0.50 x + y (1 - e-0.75 x), 0.25 - 0.25 y + y e-0.75 x]
> SSSgN(%, [x, y]);
[0.1478446271, 0.7043107460]
>
> AllenSIR( (.9, .3, .3, x, y), [x, y]);
[0.4 x + y (1 - e-0.9 x), 0.3 - 0.3 y + y e-0.9 x]
> SSSgN(%, [x, y]);
[0.1445190174, 0.7109619652]
>
> AllenSIR( (1.05, .35, .35, x, y), [x, y]);
[0.30 x + y (1 - e-1.05 x), 0.35 - 0.35 y + y e-1.05 x]
> SSSgN(%, [x, y]);
[0.1413236063, 0.7173527873]
> AllenSIR( (1.2, .4, .4, x, y), [x, y]);
[0.2 x + y (1 - e-1.2 x), 0.4 - 0.4 y + y e-1.2 x]
> SSSgN(%, [x, y]);
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
>

```

04

```

> AllenSIR( (.4, .1, .1, x, y), [x, y]);
[0.8 x + y (1 - e-0.4 x), 0.1 - 0.1 y + y e-0.4 x]
> SSSgN(%, [x, y]);
[0.2379155707, 0.5241688586]
> AllenSIR( (.6, .15, .15, x, y), [x, y]);
[0.70 x + y (1 - e-0.6 x), 0.15 - 0.15 y + y e-0.6 x]
> SSSgN(%, [x, y]);
[0.2321821549, 0.5356356903]
> AllenSIR( (.8, .2, .2, x, y), [x, y]);
[0.6 x + y (1 - e-0.8 x), 0.2 - 0.2 y + y e-0.8 x]
> SSSgN(%, [x, y]);
[0.2266503964, 0.5466992072]
>
> AllenSIR( (1, .25, .25, x, y), [x, y]);
[0.50 x + y (1 - e-x), 0.25 - 0.25 y + y e-x]
> SSSgN(%, [x, y]);
[0.2213159112, 0.5573681774]
>
> AllenSIR( (1.2, .3, .3, x, y), [x, y]);
[0.4 x + y (1 - e-1.2 x), 0.3 - 0.3 y + y e-1.2 x]
> SSSgN(%, [x, y]);
[0.2161735994, 0.5676528015]
>
> AllenSIR( (1.4, .35, .35, x, y), [x, y]);
[0.30 x + y (1 - e-1.4 x), 0.35 - 0.35 y + y e-1.4 x]
> SSSgN(%, [x, y]);
[0.2112178323, 0.5775643353]
> AllenSIR( (1.6, .4, .4, x, y), [x, y]);
[0.2 x + y (1 - e-1.6 x), 0.4 - 0.4 y + y e-1.6 x]
> SSSgN(%, [x, y]);
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
>

```

05

```

> AllenSIR( (2, .1 , .1, x, y), [x, y]);
[0.8 x + y (1 - e-2x), 0.1 - 0.1 y + y e-2x]
> SSSgN(%, [x, y]);
[0.4257286974, 0.1485426054]
> AllenSIR( (3, .15 , .15, x, y), [x, y]);
[0.70 x + y (1 - e-3x), 0.15 - 0.15 y + y e-3x]
> SSSgN(%, [x, y]);
[0.4128067433, 0.1743865134]
> AllenSIR( (4, .2 , .2, x, y), [x, y]);
[0.6 x + y (1 - e-4x), 0.2 - 0.2 y + y e-4x]
> SSSgN(%, [x, y]);
[0.3997932150, 0.2004135702]
>
> AllenSIR( (5, .25 , .25, x, y), [x, y]);
[0.50 x + y (1 - e-5x), 0.25 - 0.25 y + y e-5x]
> SSSgN(%, [x, y]);
[0.3869316911, 0.2261366179]
>
> AllenSIR( (6, .3 , .3, x, y), [x, y]);
[0.4 x + y (1 - e-6x), 0.3 - 0.3 y + y e-6x]
> SSSgN(%, [x, y]);
[0.3743947189, 0.2512105622]
>
> AllenSIR( (7, .35 , .35, x, y), [x, y]);
[0.30 x + y (1 - e-7x), 0.35 - 0.35 y + y e-7x]
> SSSgN(%, [x, y]);
[0.3622938556, 0.2754122889]
> AllenSIR( (8, .4 , .4, x, y), [x, y]);
[0.2 x + y (1 - e-8x), 0.4 - 0.4 y + y e-8x]
> SSSgN(%, [x, y]);
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
> L

```

06

```

> AllenSIR( (20, .1 , .1, x, y), [x, y]);
[0.8 x + y (1 - e-20x), 0.1 - 0.1 y + y e-20x]
> SSSgN(%, [x, y]);
[0.4545407977, 0.09091840559]
> AllenSIR( (30, .15 , .15, x, y), [x, y]);
[0.70 x + y (1 - e-30x), 0.15 - 0.15 y + y e-30x]
> SSSgN(%, [x, y]);
[0.4347824861, 0.1304350281]
> AllenSIR( (40, .2 , .2, x, y), [x, y]);
[0.6 x + y (1 - e-40x), 0.2 - 0.2 y + y e-40x]
> SSSgN(%, [x, y]);
[0.4166666628, 0.1666666747]
>
> AllenSIR( (50, .25 , .25, x, y), [x, y]);
[0.50 x + y (1 - e-50x), 0.25 - 0.25 y + y e-50x]
> SSSgN(%, [x, y]);
[0.3999999999, 0.2000000003]
>
> AllenSIR( (60, .3 , .3, x, y), [x, y]);
[0.4 x + y (1 - e-60x), 0.3 - 0.3 y + y e-60x]
> SSSgN(%, [x, y]);
[0.3846153847, 0.2307692308]
>
> AllenSIR( (70, .35 , .35, x, y), [x, y]);
[0.30 x + y (1 - e-70x), 0.35 - 0.35 y + y e-70x]
> SSSgN(%, [x, y]);
[0.3703703703, 0.2592592593]
> AllenSIR( (80, .4 , .4, x, y), [x, y]);
[0.2 x + y (1 - e-80x), 0.4 - 0.4 y + y e-80x]
> SSSgN(%, [x, y]);
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
> L

```