Homework for Lecture 16 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Nov. 3, 2025.

Subject: hw16

with an attachment hw16FirstLast.pdf

1. For the generalized Hardy-Weinberg model given by HWg(u,v,M) where M is a 3 by 3 preferance matrix

Type

```
M:=RandMat(3,30); T:=HWg(u,v,M); SSSgN(T,[u,v]);
```

40 times and record in how many cases only one of the alleles AA, Aa, aa will survive (i.e. the output is cloase to [0,1], [1,0], or [0,0]) and in how many of them they wull also survive.

2. for each of b=c=0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4

Using

```
SSSgN(AllenSIR(a,b,c,x,y),[x,y]);
```

for a=(b+c)/2 , a=(b+c), a=1.5*(b+c), a=2*(b+c), a=10*(b+c), a=100*(b+c)

Confirm, in each case, that in the Linda Allen model, for $a = (b + c)/2$ in the long run there are no infected individuals, but after $a = (b + c)$, they will start showing up. Also confirm that even for large a, they will not all be infected but x_n and y_n tend to some number.

PROBLEM 1 :

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[29, 23, 15], [26, 13, 6], [2, 27, 12]]$$

$$T := \left[\frac{116\,u^2 + 98\,uv + 13\,v^2}{4\,(24\,u^2 + 23\,uv - 8\,v^2 - 7\,u + 9\,v + 12)}, -\frac{34\,u^2 + 18\,uv + 20\,v^2 - 34\,u - 33\,v}{2\,(24\,u^2 + 23\,uv - 8\,v^2 - 7\,u + 9\,v + 12)}\right]$$

$$[0.9999999992, 7.999999985 \times 10^{-10}]$$ Aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[14, 7, 28], [7, 3, 14], [29, 15, 15]]$$

$$T := \left[-\frac{56\,u^2 + 28\,uv + 3\,v^2}{4\,(28\,u^2 + 42\,uv + 11\,v^2 - 27\,u + v - 15)}, \frac{114\,u^2 + 129\,uv + 26\,v^2 - 114\,u - 29\,v}{2\,(28\,u^2 + 42\,uv + 11\,v^2 - 27\,u + v - 15)}\right]$$

$$[5.085632845 \times 10^{-34}, 9.749093655 \times 10^{-17}]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[8, 9, 17], [26, 18, 11], [20, 23, 13]]$$

$$T := \left[-\frac{16\,u^2 + 35\,uv + 9\,v^2}{2\,(16\,u^2 + 10\,uv + 3\,v^2 - 11\,u - 8\,v - 13)}, \frac{74\,u^2 + 73\,uv + 16\,v^2 - 74\,u - 34\,v}{2\,(16\,u^2 + 10\,uv + 3\,v^2 - 11\,u - 8\,v - 13)}\right]$$

$$[0.1986145182, 0.5214197860]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[11, 5, 16], [11, 4, 27], [22, 8, 26]]$$

$$T := \left[-\frac{11\,u^2 + 8\,uv + v^2}{u^2 + 5\,uv + 5\,v^2 + 14\,u + 17\,v - 26}, \frac{76\,u^2 + 95\,uv + 31\,v^2 - 76\,u - 35\,v}{2\,(u^2 + 5\,uv + 5\,v^2 + 14\,u + 17\,v - 26)}\right]$$

$$[1.314787653 \times 10^{-341}, 1.244454961 \times 10^{-170}]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[27, 7, 2], [12, 17, 5], [26, 3, 15]]$$

$$T := \left[\frac{108\,u^2 + 38\,uv + 17\,v^2}{4\,(14\,u^2 + 13\,uv + 24\,v^2 - 2\,u - 22\,v + 15)}, -\frac{56\,u^2 + 45\,uv - 9\,v^2 - 56\,u - 8\,v}{2\,(14\,u^2 + 13\,uv + 24\,v^2 - 2\,u - 22\,v + 15)}\right]$$

$$[1.000000000, -6.511136825 \times 10^{-817}]$$ Aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[10, 17, 28], [5, 25, 1], [30, 1, 19]]$$

$$T := \left[-\frac{40\,u^2 + 44\,uv + 25\,v^2}{4\,(29\,u^2 - 42\,v^2 - 20\,u + 36\,v - 19)}, \frac{116\,u^2 + 96\,uv - 23\,v^2 - 116\,u - 2\,v}{2\,(29\,u^2 - 42\,v^2 - 20\,u + 36\,v - 19)}\right]$$

$$[0.2956610970, 0.5451146075]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[17, 28, 29], [20, 15, 24], [5, 17, 21]]$$

$$T := \left[\frac{68\,u^2 + 96\,uv + 15\,v^2}{4\,(4\,u^2 + 15\,uv - 5\,v^2 - 8\,u - v + 21)}, -\frac{68\,u^2 + 61\,uv + 26\,v^2 - 68\,u - 41\,v}{2\,(4\,u^2 + 15\,uv - 5\,v^2 - 8\,u - v + 21)}\right]$$

$$[0.5500928410, 0.3949476804]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[25, 29, 2], [2, 12, 28], [19, 21, 4]]$$

$$T := \left[\frac{50\,u^2 + 31\,uv + 6\,v^2}{2\,(8\,u^2 - 31\,uv - 33\,v^2 + 13\,u + 41\,v + 4)}, -\frac{42\,u^2 + 60\,uv + 37\,v^2 - 42\,u - 49\,v}{2\,(8\,u^2 - 31\,uv - 33\,v^2 + 13\,u + 41\,v + 4)}\right]$$

$$[1.000000000, 7.914219430 \times 10^{-19}]$$ Aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[3, 8, 16], [9, 9, 14], [14, 21, 3]]$$

$$T := \left[-\frac{12\,u^2 + 34\,uv + 9\,v^2}{4\,(24\,u^2 + 42\,uv + 23\,v^2 - 24\,u - 29\,v - 3)}, \frac{60\,u^2 + 78\,uv + 26\,v^2 - 60\,u - 35\,v}{2\,(24\,u^2 + 42\,uv + 23\,v^2 - 24\,u - 29\,v - 3)}\right]$$

$$[0.08536612952, 0.5219484615]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[28, 10, 17], [14, 12, 26], [23, 15, 7]]$$

$$T := \left[-\frac{28\,u^2 + 12\,uv + 3\,v^2}{5\,u^2 + 43\,uv + 22\,v^2 - 26\,u - 27\,v - 7}, \frac{80\,u^2 + 97\,uv + 29\,v^2 - 80\,u - 41\,v}{2\,(5\,u^2 + 43\,uv + 22\,v^2 - 26\,u - 27\,v - 7)}\right]$$

$$[1.000000000, -6.972498950 \times 10^{-141}]$$ Aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[24, 14, 13], [22, 2, 24], [5, 27, 15]]$$

$$T := \left[\frac{48u^2 + 36uv + v^2}{2(21u^2 - 3uv - 34v^2 - 12u + 21v + 15)}, -\frac{36u^2 + 51uv + 49v^2 - 36u - 51v}{2(21u^2 - 3uv - 34v^2 - 12u + 21v + 15)}\right]$$

$$[0.003632274242, 0.3031559217]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[10, 25, 11], [12, 4, 12], [15, 30, 16]]$$

$$T := \left[\frac{20u^2 + 37uv + 2v^2}{2(uv - 22v^2 - 6u + 10v + 16)}, -\frac{52u^2 + 57uv + 38v^2 - 52u - 42v}{2(uv - 22v^2 - 6u + 10v + 16)}\right]$$

$$[0.003807647852, 0.2221871492]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[19, 15, 26], [10, 9, 30], [30, 27, 18]]$$

$$T := \left[-\frac{76u^2 + 50uv + 9v^2}{4(19u^2 + 52uv + 30v^2 - 20u - 21v - 18)}, \frac{112u^2 + 144uv + 48v^2 - 112u - 57v}{2(19u^2 + 52uv + 30v^2 - 20u - 21v - 18)}\right]$$

$$[0.01517208205, 0.3401869112]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[25, 2, 9], [5, 26, 23], [9, 18, 23]]$$

$$T := \left[\frac{50u^2 + 7uv + 13v^2}{2(30u^2 - 6uv + 8v^2 - 28u - 5v + 23)}, -\frac{36u^2 + 70uv + 15v^2 - 36u - 41v}{2(30u^2 - 6uv + 8v^2 - 28u - 5v + 23)}\right]$$

$$[1.000000000, 1.098901100 \times 10^{-10}]$$ Aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

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$$M := [[25, 11, 30], [28, 26, 24], [8, 2, 4]]$$

$$T := \left[-\frac{50u^2 + 39uv + 13v^2}{2(9u^2 + 17uv - 4v^2 - 30u - 18v - 4)}, \frac{76u^2 + 63uv - 76u - 26v}{2(9u^2 + 17uv - 4v^2 - 30u - 18v - 4)}\right]$$

$$[0.9999999980, 1.668209596 \times 10^{-9}]$$ Aa

```
> |
```

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[22, 21, 25], [29, 30, 1], [20, 11, 10]]$$

$$T := \left[-\frac{44u^2 + 50uv + 15v^2}{2(13u^2 - 13uv - 28v^2 - 25u + 8v - 10)}, \frac{45u^2 + 26uv - 9v^2 - 45u - 6v}{13u^2 - 13uv - 28v^2 - 25u + 8v - 10}\right]$$

$$[0.5807904580, 0.3704339174]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[19, 8, 8], [14, 27, 15], [7, 11, 5]]$$

$$T := \left[\frac{76u^2 + 44uv + 27v^2}{4(9u^2 - 9uv + 6v^2 + 5u + 16v + 5)}, -\frac{30u^2 + 34uv - v^2 - 30u - 26v}{2(9u^2 - 9uv + 6v^2 + 5u + 16v + 5)}\right]$$

$$[0.9999999992, 5.114246440 \times 10^{-10}]$$ Aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[9, 21, 15], [16, 11, 25], [10, 29, 28]]$$

$$T := \left[\frac{36u^2 + 74uv + 11v^2}{4(12u^2 + 14uv - 15v^2 - 31u - 2v + 28)}, -\frac{50u^2 + 67uv + 43v^2 - 50u - 54v}{2(12u^2 + 14uv - 15v^2 - 31u - 2v + 28)}\right]$$

$$[1.256121508 \times 10^{-35}, 1.090521488 \times 10^{-17}]$$ aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[22, 8, 10], [4, 21, 19], [8, 11, 1]]$$

$$T := \left[\frac{88u^2 + 24uv + 21v^2}{4(5u^2 - 34uv - 8v^2 + 16u + 28v + 1)}, -\frac{3(12u^2 + 18uv + 3v^2 - 12u - 10v)}{2(5u^2 - 34uv - 8v^2 + 16u + 28v + 1)}\right]$$

$$[1.000000000, -4.054054054 \times 10^{-10}]$$ Aa

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

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$$M := [[1, 20, 28], [28, 26, 7], [8, 12, 28]]$$

$$T := \left[-\frac{2u^2 + 48uv + 13v^2}{2(7u^2 - 49uv - 35v^2 + 20u + 37v - 28)}, \frac{72u^2 + 43uv - 7v^2 - 72u - 19v}{2(7u^2 - 49uv - 35v^2 + 20u + 37v - 28)}\right]$$

$$[2.295494702 \times 10^{-939}, -3.373852241 \times 10^{-470}]$$ aa

```
> |
```

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[7, 22, 22], [23, 2, 17], [5, 24, 16]]$$

$$T := \left[-\frac{14\,u^2 + 45\,uv + v^2}{2\,(4\,u^2 - 9\,uv + 23\,v^2 + 5\,u - 9\,v - 16)}, \frac{54\,u^2 + 50\,uv + 39\,v^2 - 54\,u - 41\,v}{2\,(4\,u^2 - 9\,uv + 23\,v^2 + 5\,u - 9\,v - 16)} \right]$$

$$[0.5427150870, 0.4361876580]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[9, 18, 29], [2, 21, 12], [3, 17, 28]]$$

$$T := \left[\frac{36\,u^2 + 40\,uv + 21\,v^2}{4\,(5\,u^2 + 15\,uv + 20\,v^2 - 24\,u - 27\,v + 28)}, -\frac{64\,u^2 + 73\,uv + 8\,v^2 - 64\,u - 29\,v}{2\,(5\,u^2 + 15\,uv + 20\,v^2 - 24\,u - 27\,v + 28)} \right]$$

$$[7.398414725 \times 10^{-572}, 3.252959610 \times 10^{-286}]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[22, 7, 22], [17, 7, 5], [4, 30, 5]]$$

$$T := \left[\frac{88\,u^2 + 48\,uv + 7\,v^2}{4\,(u^2 - 27\,uv - 23\,v^2 + 16\,u + 25\,v + 5)}, -\frac{52\,u^2 + 63\,uv + 28\,v^2 - 52\,u - 35\,v}{2\,(u^2 - 27\,uv - 23\,v^2 + 16\,u + 25\,v + 5)} \right]$$

$$[1.000000000, 4.708828564 \times 10^{-106}]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[3, 12, 27], [6, 29, 4], [27, 20, 13]]$$

$$T := \left[-\frac{12\,u^2 + 36\,uv + 29\,v^2}{4\,(38\,u^2 + 34\,uv - 18\,v^2 - 28\,u + 2\,v - 13)}, \frac{108\,u^2 + 114\,uv - 5\,v^2 - 108\,u - 24\,v}{2\,(38\,u^2 + 34\,uv - 18\,v^2 - 28\,u + 2\,v - 13)} \right]$$

$$[0.1733330917, 0.5428410970]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[21, 20, 19], [25, 7, 4], [5, 25, 3]]$$

$$T := \left[-\frac{84\,u^2 + 90\,uv + 7\,v^2}{4\,(2\,uv + 19\,v^2 - 18\,u - 23\,v - 3)}, \frac{48\,u^2 + 32\,uv + 22\,v^2 - 48\,u - 29\,v}{2\,(2\,uv + 19\,v^2 - 18\,u - 23\,v - 3)} \right]$$

$$[0.9189037440, 0.08052747750]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[6, 12, 2], [8, 6, 19], [16, 13, 5]]$$

$$T := \left[-\frac{12\,u^2 + 20\,uv + 3\,v^2}{2\,(7\,u^2 + 20\,uv + 21\,v^2 - 8\,u - 22\,v - 5)}, \frac{18\,u^2 + 24\,uv + 13\,v^2 - 18\,u - 16\,v}{7\,u^2 + 20\,uv + 21\,v^2 - 8\,u - 22\,v - 5} \right]$$

$$[0.06035227230, 0.4711966998]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[27, 14, 22], [6, 24, 30], [23, 30, 26]]$$

$$T := \left[\frac{27\,u^2 + 10\,uv + 6\,v^2}{8\,u^2 - 33\,uv - 10\,v^2 - 7\,u + 8\,v + 26}, -\frac{45\,u^2 + 65\,uv + 18\,v^2 - 45\,u - 30\,v}{8\,u^2 - 33\,uv - 10\,v^2 - 7\,u + 8\,v + 26} \right]$$

$$[1.000000000, -4.652850404 \times 10^{-11}]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[13, 15, 16], [16, 6, 22], [22, 10, 6]]$$

$$T := \left[-\frac{26\,u^2 + 31\,uv + 3\,v^2}{2\,(19\,u^2 + 27\,uv + 20\,v^2 - 26\,u - 20\,v - 6)}, \frac{76\,u^2 + 77\,uv + 26\,v^2 - 76\,u - 32\,v}{2\,(19\,u^2 + 27\,uv + 20\,v^2 - 26\,u - 20\,v - 6)} \right]$$

$$[0.7548302220, 0.2365637790]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[7, 24, 25], [16, 4, 13], [29, 27, 3]]$$

$$T := \left[-\frac{7\,u^2 + 20\,uv + v^2}{44\,u^2 + 48\,uv + 33\,v^2 - 48\,u - 34\,v - 3}, \frac{2\,(27\,u^2 + 27\,uv + 9\,v^2 - 27\,u - 10\,v)}{44\,u^2 + 48\,uv + 33\,v^2 - 48\,u - 34\,v - 3} \right]$$

$$[0.3833021014, 0.5280305164]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[2, 11, 5], [21, 20, 8], [20, 6, 4]]$$

$$T := \left[-\frac{2\,u^2 + 16\,uv + 5\,v^2}{19\,u^2 - uv - 10\,v^2 - 17\,u - 6\,v - 4}, \frac{25\,u^2 + 16\,uv - 3\,v^2 - 25\,u - 7\,v}{19\,u^2 - uv - 10\,v^2 - 17\,u - 6\,v - 4} \right]$$

$$[0.3146199747, 0.5331476283]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[29, 9, 24], [18, 5, 26], [22, 12, 22]]$$

$$T := \left[\frac{116\,u^2 + 54\,uv + 5\,v^2}{4\,(5\,u^2 - 13\,uv - 11\,v^2 + 2\,u - 6\,v + 22)}, -\frac{92\,u^2 + 103\,uv + 33\,v^2 - 92\,u - 38\,v}{2\,(5\,u^2 - 13\,uv - 11\,v^2 + 2\,u - 6\,v + 22)}\right]$$

$$[1.865191270 \times 10^{-130}, 4.948219328 \times 10^{-65}]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[12, 12, 13], [24, 12, 11], [1, 17, 22]]$$

$$T := \left[\frac{3\,(4\,u^2 + 6\,uv + v^2)}{2\,(10\,u^2 + 19\,uv + 3\,v^2 - 15\,u - 8\,v + 11)}, -\frac{7\,u^2 + 5\,uv + 4\,v^2 - 7\,u - 7\,v}{10\,u^2 + 19\,uv + 3\,v^2 - 15\,u - 8\,v + 11}\right]$$

$$[2.515179309 \times 10^{-394}, -2.733005740 \times 10^{-197}]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[2, 12, 24], [24, 27, 13], [29, 22, 2]]$$

$$T := \left[-\frac{8\,u^2 + 72\,uv + 27\,v^2}{4\,(49\,u^2 + 48\,uv + 6\,v^2 - 49\,u - 31\,v - 2)}, \frac{106\,u^2 + 105\,uv + 8\,v^2 - 106\,u - 35\,v}{2\,(49\,u^2 + 48\,uv + 6\,v^2 - 49\,u - 31\,v - 2)}\right]$$

$$[0.2236131392, 0.5595034095]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[13, 12, 14], [1, 13, 7], [2, 20, 15]]$$

$$T := \left[\frac{13\,(4\,u^2 + 2\,uv + v^2)}{4\,(12\,u^2 + v^2 - 14\,u - 3\,v + 15)}, -\frac{32\,u^2 + 46\,uv + 14\,v^2 - 32\,u - 27\,v}{2\,(12\,u^2 + v^2 - 14\,u - 3\,v + 15)}\right]$$

$$[1.000000000, -1.064734846 \times 10^{-10}]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[14, 6, 26], [21, 6, 12], [15, 26, 9]]$$

$$T := \left[-\frac{28\,u^2 + 27\,uv + 3\,v^2}{2\,(18\,u^2 + 34\,uv + 23\,v^2 - 23\,u - 20\,v - 9)}, \frac{82\,u^2 + 93\,uv + 32\,v^2 - 82\,u - 38\,v}{2\,(18\,u^2 + 34\,uv + 23\,v^2 - 23\,u - 20\,v - 9)}\right]$$

$$[0.04202599244, 0.4445877674]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[25, 1, 24], [24, 9, 19], [13, 8, 29]]$$

$$T := \left[\frac{100\,u^2 + 50\,uv + 9\,v^2}{4\,(17\,u^2 + 19\,uv + 11\,v^2 - 21\,u - 31\,v + 29)}, -\frac{74\,u^2 + 76\,uv + 18\,v^2 - 74\,u - 27\,v}{2\,(17\,u^2 + 19\,uv + 11\,v^2 - 21\,u - 31\,v + 29)}\right]$$

$$[2.207112136 \times 10^{-664}, 2.482879884 \times 10^{-332}]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[5, 21, 27], [23, 8, 16], [29, 3, 10]]$$

$$T := \left[-\frac{5\,u^2 + 22\,uv + 2\,v^2}{41\,u^2 + 11\,uv + v^2 - 36\,u + v - 10}, \frac{112\,u^2 + 87\,uv + 11\,v^2 - 112\,u - 19\,v}{2\,(41\,u^2 + 11\,uv + v^2 - 36\,u + v - 10)}\right]$$

$$[0.4282268290, 0.5144539795]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[29, 15, 18], [2, 13, 2], [5, 5, 29]]$$

$$T := \left[\frac{116\,u^2 + 34\,uv + 13\,v^2}{4\,(35\,u^2 + 45\,uv + 35\,v^2 - 35\,u - 51\,v + 29)}, -\frac{46\,u^2 + 36\,uv - 6\,v^2 - 46\,u - 7\,v}{2\,(35\,u^2 + 45\,uv + 35\,v^2 - 35\,u - 51\,v + 29)}\right]$$

$$[3.934369625 \times 10^{-1831}, 2.261335205 \times 10^{-916}]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

$$M := [[28, 9, 21], [8, 8, 15], [16, 9, 8]]$$

$$T := \left[-\frac{56\,u^2 + 17\,uv + 4\,v^2}{2\,(u^2 + 28\,uv + 8\,v^2 - 21\,u - 8\,v - 8)}, \frac{74\,u^2 + 81\,uv + 16\,v^2 - 74\,u - 24\,v}{2\,(u^2 + 28\,uv + 8\,v^2 - 21\,u - 8\,v - 8)}\right]$$

$$[1.000000000, 1.250000000 \times 10^{-10}]$$

```
> M := RandMat(3, 30); T := HWg(u, v, M); SSSgN(T, [u, v]);
```

Page Break

$$M := [[3, 26, 12], [10, 19, 7], [29, 23, 5]]$$

$$T := \left[-\frac{12\,u^2 + 72\,uv + 19\,v^2}{4\,(33\,u^2 + 25\,uv + 6\,v^2 - 31\,u - 20\,v - 5)}, \frac{82\,u^2 + 76\,uv + 11\,v^2 - 82\,u - 30\,v}{2\,(33\,u^2 + 25\,uv + 6\,v^2 - 31\,u - 20\,v - 5)}\right]$$

$$[0.2489565087, 0.5504940160]$$

Problem 2

A = (b+c)/2

```
> b := 0.1;
```

$$b := 0.1$$

```
> c := 0.1;
```

$$c := 0.1$$

```
> a := (b + c)/2;
```

$$a := 0.1000000000$$

```
> SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$[7. \times 10^{-10}, 0.9999999986]$$

```
> b := 0.15 : c := 0.15: a := (b + c)/2;
```

$$b := 0.15$$
$$c := 0.15$$
$$a := 0.1500000000$$

```
> SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$[4. \times 10^{-10}, 0.9999999991]$$

```
> b := 0.2 : c := 0.2: a := (b + c)/2;
```

$$b := 0.2$$
$$c := 0.2$$
$$a := 0.2000000000$$

```
> SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$[3. \times 10^{-10}, 0.9999999993]$$

```
> b := 0.25 : c := 0.25: a := (b + c)/2;
```

$$b := 0.25$$
$$c := 0.25$$
$$a := 0.2500000000$$

```
> SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$[0., 0.9999999998]$$

```
> b := 0.3 : c := 0.3: a := (b + c)/2;
```

$$b := 0.3$$
$$c := 0.3$$
$$a := 0.3000000000$$

```
> SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$[2. \times 10^{-10}, 0.9999999996]$$

```
> b := 0.35 ; c := 0.35; a := (b + c)/2;
```

$$b := 0.35$$
$$c := 0.35$$
$$a := 0.3500000000$$

```
> SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$[2. \times 10^{-10}, 0.9999999996]$$

```
> b := 0.4 ; c := 0.4; a := (b + c)/2;
```

$$b := 0.4$$
$$c := 0.4$$
$$a := 0.4000000000$$

```
> SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
```

A = b+c

```
> a := b + c
```

$$a := b + c$$

```
> b := 0.1; c := 0.1;   SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.1$$
$$c := 0.1$$
$$[0.0023458826, 0.9952845412]$$

```
> b := 0.15; c := 0.15;   SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.15$$
$$c := 0.15$$
$$[0.0015293689, 0.9969310399]$$

```
> b := 0.2; c := 0.2;   SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.2$$
$$c := 0.2$$
$$[0.0011217441, 0.9977509080]$$

```
> b := 0.25; c := 0.25;   SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.25$$
$$c := 0.25$$
$$[0.0008777887, 0.9982409222]$$

```
> b := 0.3; c := 0.3;   SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.3$$
$$c := 0.3$$
$$[0.0007156888, 0.9985662474]$$

```
> b := 0.35; c := 0.35;   SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.35$$
$$c := 0.35$$
$$[0.0006012272, 0.9987958353]$$

```
> b := 0.4; c := 0.4;   SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.4$$
$$c := 0.4$$

```
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
```

A = 1.5*(b+c)

```
> a := 1.5*(b + c);
```

$$a := 1.20$$

```
> b := 0.1; c := 0.1;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.1$$
$$c := 0.1$$
$$[0.3953424208, 0.2093151584]$$

```
> b := 0.15; c := 0.15;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.15$$
$$c := 0.15$$
$$[0.3471603746, 0.3056792510]$$

```
> b := 0.2; c := 0.2;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.2$$
$$c := 0.2$$
$$[0.3013824127, 0.3972351747]$$

```
> b := 0.25; c := 0.25;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.25$$
$$c := 0.25$$
$$[0.2577848748, 0.4844302502]$$

```
> b := 0.3; c := 0.3;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.3$$
$$c := 0.3$$
$$[0.2161735994, 0.5676528015]$$

```
> b := 0.35; c := 0.35;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.35$$
$$c := 0.35$$
$$[0.1763789884, 0.6472420232]$$

```
> b := 0.4; c := 0.4;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.4$$
$$c := 0.4$$

```
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
```

A = 2*(b+c)

```
> a := 2·(b + c);
```

$$a := 1.6$$

```
> b := 0.1; c := 0.1;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.1$$
$$c := 0.1$$
$$[0.4145007688, 0.1709984624]$$

```
> b := 0.15; c := 0.15;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.15$$
$$c := 0.15$$
$$[0.3753020949, 0.2493958100]$$

```
> b := 0.2; c := 0.2;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.2$$
$$c := 0.2$$
$$[0.3381505031, 0.3236989942]$$

```
> b := 0.25; c := 0.25;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.25$$
$$c := 0.25$$
$$[0.3028485824, 0.3943028352]$$

```
> b := 0.3; c := 0.3;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.3$$
$$c := 0.3$$
$$[0.2692257622, 0.4615484756]$$

```
> b := 0.35; c := 0.35;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.35$$
$$c := 0.35$$
$$[0.2371336990, 0.5257326021]$$

```
> b := 0.4; c := 0.4;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.4$$
$$c := 0.4$$

```
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
```

A = 10*(b+c)

```
> a := 10 · (b + c);
```

$$a := 8.0$$

```
> b := 0.1; c := 0.1;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.1$$
$$c := 0.1$$
$$[0.4534196347, 0.09316073135]$$

```
> b := 0.15; c := 0.15;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.15$$
$$c := 0.15$$
$$[0.4329569948, 0.1340860107]$$

```
> b := 0.2; c := 0.2;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.2$$
$$c := 0.2$$
$$[0.4140578686, 0.1718842632]$$

```
> b := 0.25; c := 0.25;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.25$$
$$c := 0.25$$
$$[0.3965309479, 0.2069381044]$$

```
> b := 0.3; c := 0.3;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.3$$
$$c := 0.3$$
$$[0.3802153687, 0.2395692628]$$

```
> b := 0.35; c := 0.35;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.35$$
$$c := 0.35$$
$$[0.3649749170, 0.2700501660]$$

```
> b := 0.4; c := 0.4;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.4$$
$$c := 0.4$$

```
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
```

A = 100*(b+c)

```
> a := 100·(b + c);
```

$$a := 80.0$$

```
> b := 0.1; c := 0.1;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.1$$
$$c := 0.1$$
$$[0.4545454547, 0.09090909091]$$

```
> b := 0.15; c := 0.15;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.15$$
$$c := 0.15$$
$$[0.4347826088, 0.1304347826]$$

```
> b := 0.2; c := 0.2;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.2$$
$$c := 0.2$$
$$[0.4166666668, 0.1666666667]$$

```
> b := 0.25; c := 0.25;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.25$$
$$c := 0.25$$
$$[0.4000000000, 0.2000000000]$$

```
> b := 0.3; c := 0.3;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.3$$
$$c := 0.3$$
$$[0.3846153847, 0.2307692308]$$

```
> b := 0.35; c := 0.35;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.35$$
$$c := 0.35$$
$$[0.3703703703, 0.2592592593]$$

```
> b := 0.4; c := 0.4;    SSSgN(AllenSIR(a, b, c, x, y), [x, y]);
```

$$b := 0.4$$
$$c := 0.4$$

```
Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|
```