

Homework for Lecture 16 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Nov. 3, 2025.

Subject: hw16

with an attachment hw16FirstLast.pdf

1. For the generalized Hardy-Weinberg model given by $HWg(u,v,M)$ where M is a 3 by 3 preference matrix

Type

```
M:=RandMat(3,30); T:=HWg(u,v,M); SSSgN(T,[u,v]);
```

40 times and record in how many cases only one of the alleles AA , Aa , aa will survive (i.e. the output is close to $[0,1]$, $[1,0]$, or $[0,0]$) and in how many of them they will also survive.

2. for each of $b=c=0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4$

Using

```
SSSgN(AllenSIR(a,b,c,x,y),[x,y]);
```

for $a=(b+c)/2$, $a=(b+c)$, $a=1.5*(b+c)$, $a=2*(b+c)$, $a=10*(b+c)$, $a=100*(b+c)$

Confirm, in each case, that in the Linda Allen model, for $a = (b + c)/2$ in the long run there are no infected individuals, but after $a = (b + c)$, they will start showing up. Also confirm that even for large a , they will not all be infected but x_n and y_n tend to some number.

> read `DMB.txt`

For a list of the Main procedures type: *Help()*; for help with a specific procedure type: *Help (ProcedureName)*; for example *Help(Feig)*;

For a list of the Continuous Dynamical Models procedures type: *HelpC()*; for help with a specific procedure type: *Help(ProcedureName)*; for example *Help(Feig)*; (1)

> #problem 1

for i from 1 to 40 do

$M := \text{RandMat}(3, 30);$

$T := \text{HWg}(u, v, M);$

$\text{SSSgN}(T, [u, v]);$

end

$M := [[14, 7, 28], [7, 3, 14], [29, 15, 15]]$

$$T := \left[-\frac{56u^2 + 28uv + 3v^2}{4(28u^2 + 42uv + 11v^2 - 27u + v - 15)}, \frac{114u^2 + 129uv + 26v^2 - 114u - 29v}{2(28u^2 + 42uv + 11v^2 - 27u + v - 15)} \right]$$

$[5.085632845 \times 10^{-34}, 9.749093655 \times 10^{-17}]$

$[0, 0]$

$M := [[8, 9, 17], [26, 18, 11], [20, 23, 13]]$

$$T := \left[-\frac{16u^2 + 35uv + 9v^2}{2(16u^2 + 10uv + 3v^2 - 11u - 8v - 13)}, \frac{74u^2 + 73uv + 16v^2 - 74u - 34v}{2(16u^2 + 10uv + 3v^2 - 11u - 8v - 13)} \right]$$

$[0.1986145182, 0.5214197860]$

$M := [[11, 5, 16], [11, 4, 27], [22, 8, 26]]$

$$T := \left[-\frac{11u^2 + 8uv + v^2}{u^2 + 5uv + 5v^2 + 14u + 17v - 26}, \frac{76u^2 + 95uv + 31v^2 - 76u - 35v}{2(u^2 + 5uv + 5v^2 + 14u + 17v - 26)} \right]$$

$[1.314787653 \times 10^{-341}, 1.244454961 \times 10^{-170}]$

$M := [[27, 7, 2], [12, 17, 5], [26, 3, 15]]$

$$T := \left[\frac{108u^2 + 38uv + 17v^2}{4(14u^2 + 13uv + 24v^2 - 2u - 22v + 15)}, -\frac{56u^2 + 45uv - 9v^2 - 56u - 8v}{2(14u^2 + 13uv + 24v^2 - 2u - 22v + 15)} \right]$$

$[1.000000000, -6.511136825 \times 10^{-817}]$

$[1, 0]$

$M := [[10, 17, 28], [5, 25, 1], [30, 1, 19]]$

$$T := \left[-\frac{40u^2 + 44uv + 25v^2}{4(29u^2 - 42v^2 - 20u + 36v - 19)}, \frac{116u^2 + 96uv - 23v^2 - 116u - 2v}{2(29u^2 - 42v^2 - 20u + 36v - 19)} \right]$$

$[0.2956610970, 0.5451146075]$

$M := [[17, 28, 29], [20, 15, 24], [5, 17, 21]]$

$$T := \left[\frac{68 u^2 + 96 u v + 15 v^2}{4 (4 u^2 + 15 u v - 5 v^2 - 8 u - v + 21)}, -\frac{68 u^2 + 61 u v + 26 v^2 - 68 u - 41 v}{2 (4 u^2 + 15 u v - 5 v^2 - 8 u - v + 21)} \right]$$

[0.5500928410, 0.3949476804]

$$M := [[25, 29, 2], [2, 12, 28], [19, 21, 4]]$$

$$T := \left[\frac{50 u^2 + 31 u v + 6 v^2}{2 (8 u^2 - 31 u v - 33 v^2 + 13 u + 41 v + 4)}, -\frac{42 u^2 + 60 u v + 37 v^2 - 42 u - 49 v}{2 (8 u^2 - 31 u v - 33 v^2 + 13 u + 41 v + 4)} \right]$$

[1.0000000000, $7.914219430 \times 10^{-19}$]

[1, 0]

$$M := [[3, 8, 16], [9, 9, 14], [14, 21, 3]]$$

$$T := \left[-\frac{12 u^2 + 34 u v + 9 v^2}{4 (24 u^2 + 42 u v + 23 v^2 - 24 u - 29 v - 3)}, \frac{60 u^2 + 78 u v + 26 v^2 - 60 u - 35 v}{2 (24 u^2 + 42 u v + 23 v^2 - 24 u - 29 v - 3)} \right]$$

[0.08536612952, 0.5219484615]

$$M := [[28, 10, 17], [14, 12, 26], [23, 15, 7]]$$

$$T := \left[-\frac{28 u^2 + 12 u v + 3 v^2}{5 u^2 + 43 u v + 22 v^2 - 26 u - 27 v - 7}, \frac{80 u^2 + 97 u v + 29 v^2 - 80 u - 41 v}{2 (5 u^2 + 43 u v + 22 v^2 - 26 u - 27 v - 7)} \right]$$

[1.0000000000, $-6.972498950 \times 10^{-141}$]

[1, 0]

$$M := [[24, 14, 13], [22, 2, 24], [5, 27, 15]]$$

$$T := \left[\frac{48 u^2 + 36 u v + v^2}{2 (21 u^2 - 3 u v - 34 v^2 - 12 u + 21 v + 15)}, -\frac{36 u^2 + 51 u v + 49 v^2 - 36 u - 51 v}{2 (21 u^2 - 3 u v - 34 v^2 - 12 u + 21 v + 15)} \right]$$

[0.003632274242, 0.3031559217]

$$M := [[10, 25, 11], [12, 4, 12], [15, 30, 16]]$$

$$T := \left[\frac{20 u^2 + 37 u v + 2 v^2}{2 (u v - 22 v^2 - 6 u + 10 v + 16)}, -\frac{52 u^2 + 57 u v + 38 v^2 - 52 u - 42 v}{2 (u v - 22 v^2 - 6 u + 10 v + 16)} \right]$$

[0.003807647852, 0.2221871492]

$$M := [[19, 15, 26], [10, 9, 30], [30, 27, 18]]$$

$$T := \left[-\frac{76 u^2 + 50 u v + 9 v^2}{4 (19 u^2 + 52 u v + 30 v^2 - 20 u - 21 v - 18)}, \frac{112 u^2 + 144 u v + 48 v^2 - 112 u - 57 v}{2 (19 u^2 + 52 u v + 30 v^2 - 20 u - 21 v - 18)} \right]$$

$$[0.01517208205, 0.3401869112]$$

$$M := [[25, 2, 9], [5, 26, 23], [9, 18, 23]]$$

$$T := \left[\frac{50 u^2 + 7 u v + 13 v^2}{2 (30 u^2 - 6 u v + 8 v^2 - 28 u - 5 v + 23)}, \right. \\ \left. - \frac{36 u^2 + 70 u v + 15 v^2 - 36 u - 41 v}{2 (30 u^2 - 6 u v + 8 v^2 - 28 u - 5 v + 23)} \right]$$

$$[1.0000000000, 1.098901100 \times 10^{-10}]$$

$$[1, 0]$$

$$M := [[25, 11, 30], [28, 26, 24], [8, 2, 4]]$$

$$T := \left[-\frac{50 u^2 + 39 u v + 13 v^2}{2 (9 u^2 + 17 u v - 4 v^2 - 30 u - 18 v - 4)}, \right. \\ \left. \frac{76 u^2 + 63 u v - 76 u - 26 v}{2 (9 u^2 + 17 u v - 4 v^2 - 30 u - 18 v - 4)} \right]$$

$$[0.9999999980, 1.668209596 \times 10^{-9}]$$

$$[1, 0]$$

$$M := [[22, 21, 25], [29, 30, 1], [20, 11, 10]]$$

$$T := \left[-\frac{44 u^2 + 50 u v + 15 v^2}{2 (13 u^2 - 13 u v - 28 v^2 - 25 u + 8 v - 10)}, \right. \\ \left. \frac{45 u^2 + 26 u v - 9 v^2 - 45 u - 6 v}{13 u^2 - 13 u v - 28 v^2 - 25 u + 8 v - 10} \right]$$

$$[0.5807904580, 0.3704339174]$$

$$M := [[19, 8, 8], [14, 27, 15], [7, 11, 5]]$$

$$T := \left[\frac{76 u^2 + 44 u v + 27 v^2}{4 (9 u^2 - 9 u v + 6 v^2 + 5 u + 16 v + 5)}, -\frac{30 u^2 + 34 u v - v^2 - 30 u - 26 v}{2 (9 u^2 - 9 u v + 6 v^2 + 5 u + 16 v + 5)} \right]$$

$$[0.9999999992, 5.114246440 \times 10^{-10}]$$

$$[1, 0]$$

$$M := [[9, 21, 15], [16, 11, 25], [10, 29, 28]]$$

$$T := \left[\frac{36 u^2 + 74 u v + 11 v^2}{4 (12 u^2 + 14 u v - 15 v^2 - 31 u - 2 v + 28)}, \right. \\ \left. - \frac{50 u^2 + 67 u v + 43 v^2 - 50 u - 54 v}{2 (12 u^2 + 14 u v - 15 v^2 - 31 u - 2 v + 28)} \right]$$

$$[1.256121508 \times 10^{-35}, 1.090521488 \times 10^{-17}]$$

$$[0, 0]$$

$$M := [[22, 8, 10], [4, 21, 19], [8, 11, 1]]$$

$$T := \left[\frac{88 u^2 + 24 u v + 21 v^2}{4 (5 u^2 - 34 u v - 8 v^2 + 16 u + 28 v + 1)}, \right. \\ \left. - \frac{3 (12 u^2 + 18 u v + 3 v^2 - 12 u - 10 v)}{2 (5 u^2 - 34 u v - 8 v^2 + 16 u + 28 v + 1)} \right]$$

$$[1.0000000000, -4.054054054 \times 10^{-10}]$$

$$[1, 0]$$

$$M := [[1, 20, 28], [28, 26, 7], [8, 12, 28]]$$

$$T := \left[-\frac{2u^2 + 48uv + 13v^2}{2(7u^2 - 49uv - 35v^2 + 20u + 37v - 28)}, \frac{72u^2 + 43uv - 7v^2 - 72u - 19v}{2(7u^2 - 49uv - 35v^2 + 20u + 37v - 28)} \right]$$

[0, 0]

$$[2.295494702 \times 10^{-939}, -3.373852241 \times 10^{-470}]$$

$$M := [[7, 22, 22], [23, 2, 17], [5, 24, 16]]$$

$$T := \left[-\frac{14u^2 + 45uv + v^2}{2(4u^2 - 9uv + 23v^2 + 5u - 9v - 16)}, \frac{54u^2 + 50uv + 39v^2 - 54u - 41v}{2(4u^2 - 9uv + 23v^2 + 5u - 9v - 16)} \right]$$

[0.5427150870, 0.4361876580]

$$M := [[9, 18, 29], [2, 21, 12], [3, 17, 28]]$$

$$T := \left[\frac{36u^2 + 40uv + 21v^2}{4(5u^2 + 15uv + 20v^2 - 24u - 27v + 28)}, -\frac{64u^2 + 73uv + 8v^2 - 64u - 29v}{2(5u^2 + 15uv + 20v^2 - 24u - 27v + 28)} \right]$$

[0, 0]

$$[7.398414725 \times 10^{-572}, 3.252959610 \times 10^{-286}]$$

$$M := [[22, 7, 22], [17, 7, 5], [4, 30, 5]]$$

$$T := \left[\frac{88u^2 + 48uv + 7v^2}{4(u^2 - 27uv - 23v^2 + 16u + 25v + 5)}, -\frac{52u^2 + 63uv + 28v^2 - 52u - 35v}{2(u^2 - 27uv - 23v^2 + 16u + 25v + 5)} \right]$$

[1.0000000000, 4.708828564 \times 10^{-106}]

$$M := [[3, 12, 27], [6, 29, 4], [27, 20, 13]]$$

$$T := \left[-\frac{12u^2 + 36uv + 29v^2}{4(38u^2 + 34uv - 18v^2 - 28u + 2v - 13)}, \frac{108u^2 + 114uv - 5v^2 - 108u - 24v}{2(38u^2 + 34uv - 18v^2 - 28u + 2v - 13)} \right]$$

[0.1733330917, 0.5428410970]

$$M := [[21, 20, 19], [25, 7, 4], [5, 25, 3]]$$

$$T := \left[-\frac{84u^2 + 90uv + 7v^2}{4(2uv + 19v^2 - 18u - 23v - 3)}, \frac{48u^2 + 32uv + 22v^2 - 48u - 29v}{2(2uv + 19v^2 - 18u - 23v - 3)} \right]$$

[0.9189037440, 0.08052747750]

$$M := [[6, 12, 2], [8, 6, 19], [16, 13, 5]]$$

$$T := \left[-\frac{12u^2 + 20uv + 3v^2}{2(7u^2 + 20uv + 21v^2 - 8u - 22v - 5)}, \right]$$

$$\left[\frac{18 u^2 + 24 u v + 13 v^2 - 18 u - 16 v}{7 u^2 + 20 u v + 21 v^2 - 8 u - 22 v - 5} \right]$$

$$[0.06035227230, 0.4711966998]$$

$$M := [[27, 14, 22], [6, 24, 30], [23, 30, 26]]$$

$$T := \left[\frac{27 u^2 + 10 u v + 6 v^2}{8 u^2 - 33 u v - 10 v^2 - 7 u + 8 v + 26}, -\frac{45 u^2 + 65 u v + 18 v^2 - 45 u - 30 v}{8 u^2 - 33 u v - 10 v^2 - 7 u + 8 v + 26} \right]$$

$$[1.000000000, -4.652850404 \times 10^{-11}]$$

$$[1, 0]$$

$$M := [[13, 15, 16], [16, 6, 22], [22, 10, 6]]$$

$$T := \left[-\frac{26 u^2 + 31 u v + 3 v^2}{2 (19 u^2 + 27 u v + 20 v^2 - 26 u - 20 v - 6)}, \frac{76 u^2 + 77 u v + 26 v^2 - 76 u - 32 v}{2 (19 u^2 + 27 u v + 20 v^2 - 26 u - 20 v - 6)} \right]$$

$$[0.7548302220, 0.2365637790]$$

$$M := [[7, 24, 25], [16, 4, 13], [29, 27, 3]]$$

$$T := \left[-\frac{7 u^2 + 20 u v + v^2}{44 u^2 + 48 u v + 33 v^2 - 48 u - 34 v - 3}, \frac{2 (27 u^2 + 27 u v + 9 v^2 - 27 u - 10 v)}{44 u^2 + 48 u v + 33 v^2 - 48 u - 34 v - 3} \right]$$

$$[0.3833021014, 0.5280305164]$$

$$M := [[2, 11, 5], [21, 20, 8], [20, 6, 4]]$$

$$T := \left[-\frac{2 u^2 + 16 u v + 5 v^2}{19 u^2 - u v - 10 v^2 - 17 u - 6 v - 4}, \frac{25 u^2 + 16 u v - 3 v^2 - 25 u - 7 v}{19 u^2 - u v - 10 v^2 - 17 u - 6 v - 4} \right]$$

$$[0.3146199747, 0.5331476283]$$

$$M := [[29, 9, 24], [18, 5, 26], [22, 12, 22]]$$

$$T := \left[\frac{116 u^2 + 54 u v + 5 v^2}{4 (5 u^2 - 13 u v - 11 v^2 + 2 u - 6 v + 22)}, -\frac{92 u^2 + 103 u v + 33 v^2 - 92 u - 38 v}{2 (5 u^2 - 13 u v - 11 v^2 + 2 u - 6 v + 22)} \right]$$

$$[1.865191270 \times 10^{-130}, 4.948219328 \times 10^{-65}]$$

$$[0, 0]$$

$$M := [[12, 12, 13], [24, 12, 11], [1, 17, 22]]$$

$$T := \left[\frac{3 (4 u^2 + 6 u v + v^2)}{2 (10 u^2 + 19 u v + 3 v^2 - 15 u - 8 v + 11)}, -\frac{7 u^2 + 5 u v + 4 v^2 - 7 u - 7 v}{10 u^2 + 19 u v + 3 v^2 - 15 u - 8 v + 11} \right]$$

$$[2.515179309 \times 10^{-394}, -2.733005740 \times 10^{-197}]$$

$$[0, 0]$$

$$M := [[2, 12, 24], [24, 27, 13], [29, 22, 2]]$$

$$T := \left[-\frac{8u^2 + 72uv + 27v^2}{4(49u^2 + 48uv + 6v^2 - 49u - 31v - 2)}, \frac{106u^2 + 105uv + 8v^2 - 106u - 35v}{2(49u^2 + 48uv + 6v^2 - 49u - 31v - 2)} \right]$$

[0.2236131392, 0.5595034095]

$$M := [[13, 12, 14], [1, 13, 7], [2, 20, 15]]$$

$$T := \left[\frac{13(4u^2 + 2uv + v^2)}{4(12u^2 + v^2 - 14u - 3v + 15)}, -\frac{32u^2 + 46uv + 14v^2 - 32u - 27v}{2(12u^2 + v^2 - 14u - 3v + 15)} \right]$$

$$[1.0000000000, -1.064734846 \times 10^{-10}]$$

$$M := [[14, 6, 26], [21, 6, 12], [15, 26, 9]]$$

$$[1, 0]$$

$$T := \left[-\frac{28u^2 + 27uv + 3v^2}{2(18u^2 + 34uv + 23v^2 - 23u - 20v - 9)}, \frac{82u^2 + 93uv + 32v^2 - 82u - 38v}{2(18u^2 + 34uv + 23v^2 - 23u - 20v - 9)} \right]$$

[0.04202599244, 0.4445877674]

$$M := [[25, 1, 24], [24, 9, 19], [13, 8, 29]]$$

$$T := \left[\frac{100u^2 + 50uv + 9v^2}{4(17u^2 + 19uv + 11v^2 - 21u - 31v + 29)}, -\frac{74u^2 + 76uv + 18v^2 - 74u - 27v}{2(17u^2 + 19uv + 11v^2 - 21u - 31v + 29)} \right]$$

$$[2.207112136 \times 10^{-664}, 2.482879884 \times 10^{-332}]$$

$$M := [[5, 21, 27], [23, 8, 16], [29, 3, 10]]$$

$$[0, 0]$$

$$T := \left[-\frac{5u^2 + 22uv + 2v^2}{41u^2 + 11uv + v^2 - 36u + v - 10}, \frac{112u^2 + 87uv + 11v^2 - 112u - 19v}{2(41u^2 + 11uv + v^2 - 36u + v - 10)} \right]$$

[0.4282268290, 0.5144539795]

$$M := [[29, 15, 18], [2, 13, 2], [5, 5, 29]]$$

$$T := \left[\frac{116u^2 + 34uv + 13v^2}{4(35u^2 + 45uv + 35v^2 - 35u - 51v + 29)}, -\frac{46u^2 + 36uv - 6v^2 - 46u - 7v}{2(35u^2 + 45uv + 35v^2 - 35u - 51v + 29)} \right]$$

$$[3.934369625 \times 10^{-1831}, 2.261335205 \times 10^{-916}]$$

$$M := [[28, 9, 21], [8, 8, 15], [16, 9, 8]]$$

$$[0, 0]$$

$$T := \left[-\frac{56u^2 + 17uv + 4v^2}{2(u^2 + 28uv + 8v^2 - 21u - 8v - 8)}, \frac{74u^2 + 81uv + 16v^2 - 74u - 24v}{2(u^2 + 28uv + 8v^2 - 21u - 8v - 8)} \right]$$

$$[1.0000000000, 1.250000000 \times 10^{-10}]$$

$$M := [[3, 26, 12], [10, 19, 7], [29, 23, 5]]$$

$$[1, 0]$$

$$T := \left[-\frac{12u^2 + 72uv + 19v^2}{4(33u^2 + 25uv + 6v^2 - 31u - 20v - 5)}, \frac{82u^2 + 76uv + 11v^2 - 82u - 30v}{2(33u^2 + 25uv + 6v^2 - 31u - 20v - 5)} \right]$$

$$[0.2489565087, 0.5504940160]$$

$$M := [[25, 7, 10], [17, 2, 20], [29, 24, 26]]$$

$$T := \left[\frac{50u^2 + 24uv + v^2}{2(12u^2 - 7uv - 16v^2 - 13u - 8v + 26)}, -\frac{39u^2 + 49uv + 21v^2 - 39u - 22v}{12u^2 - 7uv - 16v^2 - 13u - 8v + 26} \right]$$

$$[2.001942190 \times 10^{-148}, 8.633298773 \times 10^{-74}] \quad [0, 0] \quad (2)$$

> #problem 2
> Help(AllenSIR)
AllenSIR(a,b,c,x,y): The Linda Allen discrete SIR model given in <https://sites.math.rutgers.edu/~zeilberg/Bio25/AllenSIR.pdf>
with parameters a,b,c. satisfying $0 < a, b, c$ and $0 < b + c \leq 1$, with initial conditions $x_0, y_0 > 0$ and $0 < x_0 + y_0 \leq 1$. Try:

$$AllenSIR(0.2, 0.3, 0.4, x, y); \quad (3)$$

> #bc_vals := [0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4]:
#a_factors := $\left[\frac{1}{2}, 1, \frac{3}{2}, 2, 10, 100 \right]$:
#a=0.5·(b+c), a=(b+c), a=1.5·(b+c), a=2·(b+c), a=10·(b+c), a=100·(b+c)
> #b=c=.1
b := 0.1 : c := 0.1 : a := 0.5(b+c);
A := AllenSIR(a, b, c, x, y);
SSSgN(A, [x, y]);
a := 0.10
A := $[0.8x + y(1 - e^{-0.10x}), 0.1 - 0.1y + ye^{-0.10x}]$
 $[7. \times 10^{-10}, 0.9999999986]$ (4)

> b := 0.1 : c := 0.1 : a := (b+c);
A := AllenSIR(a, b, c, x, y);
SSSgN(A, [x, y]);
a := 0.2
A := $[0.8x + y(1 - e^{-0.2x}), 0.1 - 0.1y + ye^{-0.2x}]$
 $[0.0023458826, 0.9952845412]$ (5)

> b := 0.1 : c := 0.1 : a := 1.5·(b+c);
A := AllenSIR(a, b, c, x, y);

$SSSgN(A, [x, y]);$

$a := 0.30$

$$A := [0.8x + y(1 - e^{-0.30x}), 0.1 - 0.1y + ye^{-0.30x}]$$
$$[0.1586702173, 0.6826595654]$$

(6)

> $b := 0.1 : c := 0.1 : a := 2 \cdot (b + c);$

$A := AllenSIR(a, b, c, x, y);$

$SSSgN(A, [x, y]);$

$a := 0.4$

$$A := [0.8x + y(1 - e^{-0.4x}), 0.1 - 0.1y + ye^{-0.4x}]$$
$$[0.2379155707, 0.5241688586]$$

(7)

> $b := 0.1 : c := 0.1 : a := 10 \cdot (b + c);$

$A := AllenSIR(a, b, c, x, y);$

$SSSgN(A, [x, y]);$

$a := 2.0$

$$A := [0.8x + y(1 - e^{-2.0x}), 0.1 - 0.1y + ye^{-2.0x}]$$
$$[0.4257286974, 0.1485426054]$$

(8)

> $b := 0.1 : c := 0.1 : a := 100 \cdot (b + c);$

$A := AllenSIR(a, b, c, x, y);$

$SSSgN(A, [x, y]);$

$a := 20.0$

$$A := [0.8x + y(1 - e^{-20.0x}), 0.1 - 0.1y + ye^{-20.0x}]$$
$$[0.4545407977, 0.09091840559]$$

(9)

> $\#b=c=.15$

$b := 0.15 : c := 0.15 : a := 0.5 \cdot (b + c);$

$A := AllenSIR(a, b, c, x, y);$

$SSSgN(A, [x, y]);$

$a := 0.150$

$$A := [0.70x + y(1 - e^{-0.150x}), 0.15 - 0.15y + ye^{-0.150x}]$$
$$[4. \times 10^{-10}, 0.9999999991]$$

(10)

> $b := 0.15 : c := 0.15 : a := (b + c);$

$A := AllenSIR(a, b, c, x, y);$

$SSSgN(A, [x, y]);$

$a := 0.30$

$$A := [0.70x + y(1 - e^{-0.30x}), 0.15 - 0.15y + ye^{-0.30x}]$$
$$[0.0015293689, 0.9969310399]$$

(11)

> $b := 0.15 : c := 0.15 : a := 1.5 \cdot (b + c);$

$A := AllenSIR(a, b, c, x, y);$

$SSSgN(A, [x, y]);$

$a := 0.450$

$$A := [0.70x + y(1 - e^{-0.450x}), 0.15 - 0.15y + ye^{-0.450x}]$$
$$[0.1549131987, 0.6901736030]$$

(12)

$$\begin{aligned}
&> b := 0.15 : c := 0.15 : a := 2 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 0.60 \\
&A := [0.70x + y(1 - e^{-0.60x}), 0.15 - 0.15y + ye^{-0.60x}] \\
&\quad [0.2321821549, 0.5356356903]
\end{aligned} \tag{13}$$

$$\begin{aligned}
&> b := 0.15 : c := 0.15 : a := 10 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 3.00 \\
&A := [0.70x + y(1 - e^{-3.00x}), 0.15 - 0.15y + ye^{-3.00x}] \\
&\quad [0.4128067433, 0.1743865134]
\end{aligned} \tag{14}$$

$$\begin{aligned}
&> b := 0.15 : c := 0.15 : a := 100 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 30.00 \\
&A := [0.70x + y(1 - e^{-30.00x}), 0.15 - 0.15y + ye^{-30.00x}] \\
&\quad [0.4347824861, 0.1304350281]
\end{aligned} \tag{15}$$

$$\begin{aligned}
&> \#b=c=.2 \\
&> b := 0.2 : c := 0.2 : a := 0.5 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 0.150 \\
&A := [0.70x + y(1 - e^{-0.150x}), 0.15 - 0.15y + ye^{-0.150x}] \\
&\quad [4. \times 10^{-10}, 0.9999999991]
\end{aligned} \tag{16}$$

$$\begin{aligned}
&> b := 0.2 : c := 0.2 : a := (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 0.4 \\
&A := [0.6x + y(1 - e^{-0.4x}), 0.2 - 0.2y + ye^{-0.4x}] \\
&\quad [0.0011217441, 0.9977509080]
\end{aligned} \tag{17}$$

$$\begin{aligned}
&> b := 0.2 : c := 0.2 : a := 1.5 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 0.60 \\
&A := [0.6x + y(1 - e^{-0.60x}), 0.2 - 0.2y + ye^{-0.60x}] \\
&\quad [0.1513070546, 0.6973858908]
\end{aligned} \tag{18}$$

$$\begin{aligned}
&> b := 0.2 : c := 0.2 : a := 2 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 0.8
\end{aligned}$$

$$A := [0.6x + y(1 - e^{-0.8x}), 0.2 - 0.2y + ye^{-0.8x}]$$

$$[0.2266503964, 0.5466992072]$$
(19)

> $b := 0.2 : c := 0.2 : a := 10 \cdot (b + c);$
 $A := \text{AllenSIR}(a, b, c, x, y);$
 $\text{SSSgN}(A, [x, y]);$

$$a := 4.0$$

$$A := [0.6x + y(1 - e^{-4.0x}), 0.2 - 0.2y + ye^{-4.0x}]$$

$$[0.3997932150, 0.2004135702]$$
(20)

> $b := 0.2 : c := 0.2 : a := 100 \cdot (b + c);$
 $A := \text{AllenSIR}(a, b, c, x, y);$
 $\text{SSSgN}(A, [x, y]);$

$$a := 40.0$$

$$A := [0.6x + y(1 - e^{-40.0x}), 0.2 - 0.2y + ye^{-40.0x}]$$

$$[0.4166666628, 0.1666666747]$$
(21)

> $\#b=c=.25$

> $b := 0.25 : c := 0.25 : a := 0.5 \cdot (b + c);$
 $A := \text{AllenSIR}(a, b, c, x, y);$
 $\text{SSSgN}(A, [x, y]);$

$$a := 0.250$$

$$A := [0.50x + y(1 - e^{-0.250x}), 0.25 - 0.25y + ye^{-0.250x}]$$

$$[0., 0.9999999998]$$
(22)

> $b := 0.25 : c := 0.25 : a := (b + c);$
 $A := \text{AllenSIR}(a, b, c, x, y);$
 $\text{SSSgN}(A, [x, y]);$

$$a := 0.50$$

$$A := [0.50x + y(1 - e^{-0.50x}), 0.25 - 0.25y + ye^{-0.50x}]$$

$$[0.0008777887, 0.9982409222]$$
(23)

> $b := 0.25 : c := 0.25 : a := 1.5 \cdot (b + c);$
 $A := \text{AllenSIR}(a, b, c, x, y);$
 $\text{SSSgN}(A, [x, y]);$

$$a := 0.750$$

$$A := [0.50x + y(1 - e^{-0.750x}), 0.25 - 0.25y + ye^{-0.750x}]$$

$$[0.1478446271, 0.7043107460]$$
(24)

> $b := 0.25 : c := 0.25 : a := 2 \cdot (b + c);$
 $A := \text{AllenSIR}(a, b, c, x, y);$
 $\text{SSSgN}(A, [x, y]);$

$$a := 1.00$$

$$A := [0.50x + y(1 - e^{-1.00x}), 0.25 - 0.25y + ye^{-1.00x}]$$

$$[0.2213159112, 0.5573681774]$$
(25)

> $b := 0.25 : c := 0.25 : a := 10 \cdot (b + c);$
 $A := \text{AllenSIR}(a, b, c, x, y);$

SSSgN(A, [x, y]);

$a := 5.00$

$A := [0.50x + y(1 - e^{-5.00x}), 0.25 - 0.25y + ye^{-5.00x}]$
 $[0.3869316911, 0.2261366179]$

(26)

> $b := 0.25 : c := 0.25 : a := 100 \cdot (b + c);$

$A := \text{AllenSIR}(a, b, c, x, y);$

SSSgN(A, [x, y]);

$a := 50.00$

$A := [0.50x + y(1 - e^{-50.00x}), 0.25 - 0.25y + ye^{-50.00x}]$
 $[0.3999999999, 0.2000000003]$

(27)

> $\#b=c=.3$

> $b := 0.3 : c := 0.3 : a := 0.5 \cdot (b + c);$

$A := \text{AllenSIR}(a, b, c, x, y);$

SSSgN(A, [x, y]);

$a := 0.30$

$A := [0.4x + y(1 - e^{-0.30x}), 0.3 - 0.3y + ye^{-0.30x}]$
 $[2. \times 10^{-10}, 0.9999999996]$

(28)

> $b := 0.3 : c := 0.3 : a := (b + c);$

$A := \text{AllenSIR}(a, b, c, x, y);$

SSSgN(A, [x, y]);

$a := 0.6$

$A := [0.4x + y(1 - e^{-0.6x}), 0.3 - 0.3y + ye^{-0.6x}]$
 $[0.0007156888, 0.9985662474]$

(29)

> $b := 0.3 : c := 0.3 : a := 1.5 \cdot (b + c);$

$A := \text{AllenSIR}(a, b, c, x, y);$

SSSgN(A, [x, y]);

$a := 0.90$

$A := [0.4x + y(1 - e^{-0.90x}), 0.3 - 0.3y + ye^{-0.90x}]$
 $[0.1445190174, 0.7109619652]$

(30)

> $b := 0.3 : c := 0.3 : a := 2 \cdot (b + c);$

$A := \text{AllenSIR}(a, b, c, x, y);$

SSSgN(A, [x, y]);

$a := 1.2$

$A := [0.4x + y(1 - e^{-1.2x}), 0.3 - 0.3y + ye^{-1.2x}]$
 $[0.2161735994, 0.5676528015]$

(31)

> $b := 0.3 : c := 0.3 : a := 10 \cdot (b + c);$

$A := \text{AllenSIR}(a, b, c, x, y);$

SSSgN(A, [x, y]);

$a := 6.0$

$A := [0.4x + y(1 - e^{-6.0x}), 0.3 - 0.3y + ye^{-6.0x}]$
 $[0.3743947189, 0.2512105622]$

(32)

$$\begin{aligned}
&> b := 0.3 : c := 0.3 : a := 100 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 60.0 \\
&A := [0.4x + y(1 - e^{-60.0x}), 0.3 - 0.3y + ye^{-60.0x}] \\
&\quad [0.3846153847, 0.2307692308]
\end{aligned} \tag{33}$$

$$\begin{aligned}
&> \#b=c=.35 \\
&> b := 0.35 : c := 0.35 : a := 0.5 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 0.350 \\
&A := [0.30x + y(1 - e^{-0.350x}), 0.35 - 0.35y + ye^{-0.350x}] \\
&\quad [2. \times 10^{-10}, 0.9999999996]
\end{aligned} \tag{34}$$

$$\begin{aligned}
&> b := 0.35 : c := 0.35 : a := (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 0.70 \\
&A := [0.30x + y(1 - e^{-0.70x}), 0.35 - 0.35y + ye^{-0.70x}] \\
&\quad [0.0006012272, 0.9987958353]
\end{aligned} \tag{35}$$

$$\begin{aligned}
&> b := 0.35 : c := 0.35 : a := 1.5 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 1.050 \\
&A := [0.30x + y(1 - e^{-1.050x}), 0.35 - 0.35y + ye^{-1.050x}] \\
&\quad [0.1413236063, 0.7173527873]
\end{aligned} \tag{36}$$

$$\begin{aligned}
&> b := 0.35 : c := 0.35 : a := 2 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 1.40 \\
&A := [0.30x + y(1 - e^{-1.40x}), 0.35 - 0.35y + ye^{-1.40x}] \\
&\quad [0.2112178323, 0.5775643353]
\end{aligned} \tag{37}$$

$$\begin{aligned}
&> b := 0.35 : c := 0.35 : a := 10 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 7.00 \\
&A := [0.30x + y(1 - e^{-7.00x}), 0.35 - 0.35y + ye^{-7.00x}] \\
&\quad [0.3622938556, 0.2754122889]
\end{aligned} \tag{38}$$

$$\begin{aligned}
&> b := 0.35 : c := 0.35 : a := 100 \cdot (b + c); \\
&A := \text{AllenSIR}(a, b, c, x, y); \\
&\text{SSSgN}(A, [x, y]); \\
&a := 70.00
\end{aligned}$$

$$A := [0.30x + y(1 - e^{-70.00x}), 0.35 - 0.35y + ye^{-70.00x}]$$

$$[0.3703703703, 0.2592592593]$$

(39)

> #b=c=.4

>

> b := 0.4 : c := 0.4 : a := 0.5 · (b + c);
 A := AllenSIR(a, b, c, x, y);
 SSSgN(A, [x, y]);

$$a := 0.40$$

$$A := [0.2x + y(1 - e^{-0.40x}), 0.4 - 0.4y + ye^{-0.40x}]$$

Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|

> b := 0.4 : c := 0.4 : a := (b + c);
 A := AllenSIR(a, b, c, x, y);
 SSSgN(A, [x, y]);

$$a := 0.8$$

$$A := [0.2x + y(1 - e^{-0.8x}), 0.4 - 0.4y + ye^{-0.8x}]$$

Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|

> b := 0.4 : c := 0.4 : a := 1.5 · (b + c);
 A := AllenSIR(a, b, c, x, y);
 SSSgN(A, [x, y]);

$$a := 1.20$$

$$A := [0.2x + y(1 - e^{-1.20x}), 0.4 - 0.4y + ye^{-1.20x}]$$

Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|

> b := 0.4 : c := 0.4 : a := 2 · (b + c);
 A := AllenSIR(a, b, c, x, y);
 SSSgN(A, [x, y]);

$$a := 1.6$$

$$A := [0.2x + y(1 - e^{-1.6x}), 0.4 - 0.4y + ye^{-1.6x}]$$

Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|

> b := 0.4 : c := 0.4 : a := 10 · (b + c);
 A := AllenSIR(a, b, c, x, y);
 SSSgN(A, [x, y]);

$$a := 8.0$$

$$A := [0.2x + y(1 - e^{-8.0x}), 0.4 - 0.4y + ye^{-8.0x}]$$

Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|

> b := 0.4 : c := 0.4 : a := 100 · (b + c);
 A := AllenSIR(a, b, c, x, y);
 SSSgN(A, [x, y]);

$$a := 80.0$$

$$A := [0.2x + y(1 - e^{-80.0x}), 0.4 - 0.4y + ye^{-80.0x}]$$

Error, (in sqrfree) argument must be a polynomial or a rational function in lam |DMB.txt:746|

>

In the Linda Allen model, when $a \leq (b + c)$, the infection dies out and no infected individuals remain in the long run. Once $a > (b + c)$, infections persist at steady levels—however, even for very large a , not everyone becomes infected, and x_n and y_n each approach constant equilibrium values.