Homework for Lecture 15 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 27, 2025.

Subject: hw15

with an attachment hw15FirstLast.pdf

1. Read and understand, and be able to reproduce without peeking (e.g. in examination conditions) the derivatin of the Hardy-Weinberg rule.

$$(u,v) \to \left(u^2 + vu + \frac{1}{4}v^2, -2vu - 2u^2 + 2u - \frac{1}{2}v^2 + v \right)$$

2. If right now, 20 percent of the polpulation have genotupe AA, 30 percent of the polpulation have genotype Aa, what is the percentage of aa genotypes (i) Right now? (ii) In the next generation? (iii) In ten generations?

3. If right now the 50 percents of the polpulation are of aA genotypes, and 30 percents of the polpulation are of aa genotypes, what is the percentage of AA genotypes (i) Right now? (ii) In the next generation? (iii) In ten generations?

4. Read and understand Linda Allen's article:

`http://sites.math.rutgers.edu/~zeilberg/Bio25/AllenSIR.pdf`

Experiment with procedure `AllenSIR(a,b,c,x,y)` for various values of a, b, c and find the ultimate behavior using `ORB`

in our Maple package:

`https://sites.math.rutgers.edu/~zeilberg/Bio25/DMB.txt` .

2. If right now, 20 percent of the polpulation have genotupe AA, 30 percent of the polpulation have genotype Aa, what is the percentage of aa genotypes (i) Right now? (ii) In the next generation? (iii) In ten generations?

u AA = 20%.

v Aa = 30%.

(i) 100 - 20 - 30 = **50% aa right now**

(ii) $$(u, v) \to \left(u^2 + vu + \frac{1}{4}v^2, -2vu - 2u^2 + 2u - \frac{1}{2}v^2 + v\right)$$

$$\left((.2)^2 + (.2)(.3) + \tfrac{1}{4}(.3)^2,\ -2(.2)(.3) - 2(.2)^2 + 2(.2) - \tfrac{1}{2}(.3)^2 + .3\right)$$

$$(.1225,\ .455)$$

$1 - .455 - .1225 = .4225$

↳ **42.25%.**

(iii) same as ii → **42.25%.**

3. If right now the 50 percents of the polpulation are of aA genotypes, and 30 percents of the polpulation are of aa genotypes, what is the percentage of AA genotypes (i) Right now? (ii) In the next generation? (iii) In ten generations?

v aA = 50%.

1-u-v aa = 30%.

(i) AA = 100 - 50 - 30 = **20%.**

(ii) $(.2, .5)$

$$(u, v) \to \left(u^2 + vu + \frac{1}{4}v^2, -2vu - 2u^2 + 2u - \frac{1}{2}v^2 + v\right)$$

$$\left(.2^2 + .2(.5) + \tfrac{1}{4}(.5)^2,\ -2(.2)(.5) - 2(.2)^2 + 2(.2) - \tfrac{1}{2}(.5)^2 + .5\right)$$

$$(.2025,\ .495)$$

$.2025$ → **20.25%**

iii) **20.25%**

```
> read("DMB.txt")
```

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig);
For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig);

```
> Help( );
```

The available procedures are: AllenSIR, AllenSIRg, ChaosStory, Feig, HW, NicholsonBailey, NicholsonBaileyM, Orb, ORB, Orbk, RecToT, RecToTs, SS, SSg, SSr, SSS, SSSg, SSSgN, RR, RT

```
> Help(AllenSIR)
```

AllenSIR(a,b,c,x,y): The Linda Allen discrete SIR model given in https://sites.math.rutgers.edu/~zeilberg/Bio25/AllenSIR.pdf
with parameters a,b,c. satisfying 0<a,b,c and 0<b+c<=1 , with initial conditions x0,y0>0 and 0<x0+y0<=1 .Try:
AllenSIR(0.2,0.3,0.4,x,y);

```
> AllenSIR(0.2, 0.3, 0.4, x, y);
```

$$[0.3\,x + y\,(1 - e^{-0.2\,x}),\ 0.3 - 0.3\,y + y\,e^{-0.2\,x}]$$

```
> AllenSIR(0.5, 0.3, 0.4, x, y);
```

$$[0.3\,x + y\,(1 - e^{-0.5\,x}),\ 0.3 - 0.3\,y + y\,e^{-0.5\,x}]$$

```
> AllenSIR(0.2, 0.7, 0.4, x, y);
```

$$[-0.1\,x + y\,(1 - e^{-0.2\,x}),\ 0.7 - 0.7\,y + y\,e^{-0.2\,x}]$$

```
> AllenSIR(0.2, 0.3, 0.8, x, y);
```

$$[-0.1\,x + y\,(1 - e^{-0.2\,x}),\ 0.3 - 0.3\,y + y\,e^{-0.2\,x}]$$

```
> ORB(AllenSIR(0.2, 0.3, 0.4, x, y), [x, y], [.5, .5], 1000, 1010);
[[0., 0.9999999999], [0., 0.9999999999], [0., 0.9999999999], [0., 0.9999999999], [0., 0.9999999999], [0., 0.9999999999], [0., 0.9999999999], [0., 0.9999999999], [0., 0.9999999999], [0., 0.9999999999]]
>
```