

Homework for Lecture 15 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 27, 2025.

Subject: hw13

with an attachment hw13FirstLast.pdf

1. Read and understand, and be able to reproduce without peeking (e.g. in examination conditions) the derivation of the Hardy-Weinberg rule.

$$(u, v) \rightarrow \left(u^2 + vu + \frac{1}{4}v^2, -2vu - 2u^2 + 2u - \frac{1}{2}v^2 + v \right)$$

2. If right now, 20 percent of the population have genotype *AA*, 30 percent of the population have genotype *Aa*, what is the percentage of *aa* genotypes (i) Right now? (ii) In the next generation? (iii) In ten generations?

i) $w = 1 - u - v$ $= 1 - .2 - .3$ $= .5 \Rightarrow 50\%$	ii) $p = u + \frac{1}{2}v$ $= .2 + .15 = .35$ $q = .65$	iii) Stays at HW proportions $w' = q^2 = .65^2 = .4225$ $\Rightarrow 42.25\%$
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3. If right now the 50 percent of the population are of *AA* genotypes, and 30 percent of the population are of *aa* genotypes, what is the percentage of *AA* genotypes (i) Right now? (ii) In the next generation? (iii) In ten generations?

i) $v = .5$ $w = .3$ $u = 1 - v - w = .2$ 20%	ii) $p = u + \frac{1}{2}v$ $= .2 + .15 = .35$ $q = .65$	iii) HW Proportions say $p^2 = .35^2 = .1225$ $\Rightarrow 12.25\%$
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4. Read and understand Linda Allen's article:

<http://sites.math.rutgers.edu/~zeilberg/Bio25/AllenSIR.pdf>

Experiment with procedure `AllenSIR(a, b, c, x, y)` for various values of *a*, *b*, *c* and find the ultimate behavior using ORB

$$R_0 = .8 < 1 \text{ orbit converged to } (0, 1) \text{ (disease-free)}$$

in our Maple package:

$$R_0 = 4 < 1 \text{ converged to } \sim (.4509, .3237) \text{ which matched the solution of the equilibrium equation.}$$

$$R_0 = 2.4 > 1 \text{ converged to } \sim (.222, .445) \text{ which agree with } R_0 \text{ threshold for endemic equilibrium characterization.}$$

<https://sites.math.rutgers.edu/~zeilberg/Bio25/DMB.txt> .