Homework for Lecture 15 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 27, 2025.

Subject: hw15

with an attachment hw15FirstLast.pdf

1. Read and understand, and be able to reproduce without peeking (e.g. in examination conditions) the derivatin of the Hardy-Weinberg rule.

$$(u,v) \to \left(u^2+vu+\frac{1}{4}v^2, -2vu-2u^2+2u-\frac{1}{2}v^2+v\right)$$

2. If right now, 20 percent of the polpulation have genotupe AA, 30 percent of the polpulation have genotype Aa, what is the percentage of aa genotypes (i) Right now? (ii) In the next generation? (iii) In ten generations?

3. If right now the 50 percents of the polpulation are of aA genotypes, and 30 percents of the polpulation are of aa genotypes, what is the percentage of AA genotypes (i) Right now? (ii) In the next generation? (iii) In ten generations?

4. Read and understand Linda Allen's article:

`http://sites.math.rutgers.edu/~zeilberg/Bio25/AllenSIR.pdf`

Experiment with procedure `AllenSIR(a,b,c,x,y)` for various values of a, b, c and find the ultimate behavior using `ORB`

in our Maple package:

`https://sites.math.rutgers.edu/~zeilberg/Bio25/DMB.txt` .

Daniyal Chaudhry HW15

1. Yes, I can replicate this derivation using the mating matrix we learned.

2. $AA = 20\% = \frac{1}{5} = u$
$Aa = 30\% = \frac{3}{10} = v$
$aa = w$

i) $\boxed{aa \text{ right now is } 50\%, \text{ or } \frac{1}{2}}$

ii) u in 1 gen: $\left(\frac{1}{5}\right)^2 + \left(\frac{3}{10}\right)\left(\frac{1}{5}\right) + \frac{1}{4}\left(\frac{3}{10}\right)^2 = \frac{1}{25} + \frac{3}{50} + \frac{9}{400} = \boxed{\frac{49}{400}}$

v in 1 gen: $-2\left(\frac{3}{10}\right)\left(\frac{1}{5}\right) - 2\left(\frac{1}{5}\right)^2 + 2\left(\frac{1}{5}\right) - \frac{1}{2}\left(\frac{3}{10}\right)^2 + \left(\frac{3}{10}\right) =$
$-\frac{3}{25} - \frac{2}{25} + \frac{2}{5} - \frac{9}{200} + \frac{3}{10} = \boxed{\frac{91}{200}}$

$1 - u - v = 1 - \frac{49}{400} - \frac{182}{400} = \frac{231}{400}$

$\boxed{\text{Thus, } aa \text{ in 1 gen} = \frac{231}{400}}$

iii) $\boxed{\text{Also } \frac{231}{400}, \text{ as it stabilizes after 1 iteration}}$

3. $aa = w = 30\% = \frac{3}{10}$
$Aa = v = 50\% = \frac{1}{2}$
$AA = u$

i) $\boxed{AA \text{ right now is } 20\%, \text{ or } 1/5}$

ii) u in 1 gen: $\left(\frac{1}{5}\right)^2 + \left(\frac{1}{2}\right)\left(\frac{1}{5}\right) + \frac{1}{4}\left(\frac{1}{2}\right)^2 = \frac{1}{25} + \frac{1}{10} + \frac{1}{16} = \frac{81}{400}$

$\boxed{\text{Thus, } AA \text{ in 1 gen is } \frac{81}{400} \text{ of the population}}$

iii) $\boxed{\text{Also } \frac{81}{400}, \text{ as it stabilizes}}$

4. I played around with the orbits & values of a, b, c, and found that when $a < b+c$, the orbit results in a fully infection free population (0,1). when $a > b+c$, the orbit stalled to result in a stabilized proportion of infected individuals which increased as a increased. This is in line with what the article says.