Homework for Lecture 13 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 20, 2025.

Subject: hw13

with an attachment hw13FirstLast.pdf

1. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3 + a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 + a_1(n)}$$

Which of them is a stable steady-state?

2. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3a_1(n) + 5a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2a_1(n) + 7a_2(n)}$$

Which of them is a stable steady-state?

1. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3 + a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 + a_1(n)}$$

Which of them is a stable steady-state?

$$(x,y) \to \left(\frac{x}{3+y}, \frac{y}{2+x}\right)$$

$x = \frac{x}{3+y}$

$x - \frac{x}{3+y} = 0$

$x\left(1 - \frac{1}{3+y}\right) = 0$

$x = 0$

$3+y = 1 \to y = -2$

$(0,-2)$

$(0,0)$

$y = \frac{y}{2+x}$

$y\left(1 - \frac{1}{2+x}\right) = 0$

$y = 0$

$x + 2 = 1$

$x = -1$

$(-1,0)$

$(-1,-2)$

$$J(x,y) = \begin{bmatrix} \frac{\partial}{\partial x}\frac{x}{3+y} & \frac{\partial}{\partial y}\frac{x}{3+y} \\ \frac{\partial}{\partial x}\frac{y}{2+x} & \frac{\partial}{\partial y}\frac{y}{2+x} \end{bmatrix} = \begin{bmatrix} \frac{3+y}{(3+y)^2} & \frac{-x}{(3+y)^2} \\ \frac{-y}{(2+x)^2} & \frac{2+x}{(2+x)^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{3+y} & \frac{-x}{(3+y)^2} \\ \frac{-y}{(2+x)^2} & \frac{1}{2+x} \end{bmatrix}$$

$(0,-2)$

$$J = \begin{bmatrix} 1-\lambda & 0 \\ 1/2 & \frac{1}{2}-\lambda \end{bmatrix}$$

$(1-\lambda)\left(\frac{1}{2}-\lambda\right)$

$\frac{1}{2} - \frac{3}{2}\lambda + \lambda^2$

$\lambda = 1, .5$ ↳ not stable

$(0,0)$

$$J = \begin{bmatrix} \frac{1}{3}-\lambda & 0 \\ 0 & \frac{1}{2}-\lambda \end{bmatrix}$$

$\left(\frac{1}{3}-\lambda\right)\left(\frac{1}{2}-\lambda\right) = 0$

$\lambda = 1/3, 1/2$ → stable

$(-1,0)$

$$J = \begin{bmatrix} \frac{1}{3}-\lambda & 1/9 \\ 0 & -1-\lambda \end{bmatrix}$$

$\lambda = 1/3, -1$ → unstable

$(-1,-2)$

$$J = \begin{bmatrix} 1-\lambda & 1 \\ 2 & 1-\lambda \end{bmatrix}$$

$\lambda = 1, 1$, unstable

2. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3a_1(n) + 5a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2a_1(n) + 7a_2(n)}$$

Which of them is a stable steady-state?

$(x,y) \to \left(\frac{x}{3x+5y}, \frac{y}{2x+7y}\right)$

$x = \frac{x}{3x+5y}$

$x\left(1 - \frac{1}{3x+5y}\right) = 0$

$x = 0$

$3x + 5y = 1$

$y = \frac{y}{2x+7y}$

$y\left(1 - \frac{1}{2x+7y}\right) = 0$

$y = 0$

$2x + 7y = 1$

$(0,0)$

$\left(\frac{1}{3}, 0\right)$

$\left(0, \frac{1}{7}\right)$

$\left(0, \frac{1}{4}\right)$

$$J = \begin{bmatrix} \frac{5y}{(3x+5y)^2} & \frac{-5x}{(3x+5y)^2} \\ \frac{-2y}{(2x+7y)^2} & \frac{2x}{(2x+7y)^2} \end{bmatrix}$$

$(0,0)$ → unstable

$\left(\frac{1}{3}, 0\right)$ stable

$\left(0, \frac{1}{7}\right)$ unstable

$\left(0, \frac{1}{4}\right)$ stable