

Homework 13 DMB Ezra Chechik

1) a. Find SS of System:

(Then
SSS)

$$a_1(n+1) = \frac{a_1(n)}{3 + a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 + a_1(n)}$$

underlying Transformation: $R^2 \rightarrow R^2$

$$(x, y) \rightarrow \left(\frac{x}{3+y}, \frac{y}{2+x} \right)$$

next, solve the system of eq.

$$x \left(1 - \frac{1}{3+y} \right) = 0 \quad \& \quad y \left(1 - \frac{1}{2+x} \right) = 0$$

$$\hookrightarrow y = 0, x = -1$$

$$\text{when } x = 0, y = 0 : (0, 0)$$

$$\text{when } y = -2, x = -1 : (-1, -2)$$

$$1 - \frac{1}{2+x} = 0 \quad \checkmark$$

$$\text{OR } \begin{aligned} 3+y &= 1 \\ y &= -2 \end{aligned}$$

SS: $(0, 0), (-1, -2)$, NOW to check stability

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$$J = \begin{bmatrix} \frac{\partial}{\partial x} \left(\frac{x}{3+y} \right) & \frac{\partial}{\partial y} \left(\frac{x}{3+y} \right) \\ \frac{\partial}{\partial x} \left(\frac{y}{2+x} \right) & \frac{\partial}{\partial y} \left(\frac{y}{2+x} \right) \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \frac{1}{3+y} & \frac{-x}{(3+y)^2} \\ \frac{-y}{(2+x)^2} & \frac{1}{2+x} \end{bmatrix} = J$$

$$J(0,0) = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix} \quad \lambda = 1/3, 1/2$$

Since $|\lambda|$ all < 1

$(0,0)$ is stable SS

$$J(-1, -2) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad \det(A - \lambda I) = 0$$

$$\rightarrow \det \begin{pmatrix} 1-\lambda & 1 \\ 2 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)^2 - 2 = 0 \rightarrow \lambda^2 - 2\lambda - 1 = 0$$

$$\lambda_1 = 1 - \sqrt{2} \quad \lambda_2 = 1 + \sqrt{2}$$

Since $|\lambda_2| > 1 \rightarrow (-1, -2)$ is Unstable

Only $(0,0)$ was stable.

2) a "

$$a_1(n+1) = \frac{a_1(n)}{3a_1(n) + 5a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2a_1(n) + 7a_2(n)}$$

Transformation $\mathbb{R}^2 \rightarrow \mathbb{R}^2$:

$$(x, y) \rightarrow \left(\frac{x}{3x+5y}, \frac{y}{2x+7y} \right)$$

SS: Solve System: I get $(0, 1/7), (1/3, 0), (2/11, 1/11)$

$$x \left(1 - \frac{1}{3x+5y} \right) = 0 \quad \& \quad y \left(1 - \frac{1}{2x+7y} \right) = 0$$

→ Both cannot be 0: $(0/0)$

$$x=0$$

$$y=0$$

$$1 = \frac{1}{7y}, y = 1/7$$

$$(0, 1/7)$$

$$1 = \frac{1}{3x}, x = 1/3$$

$$(1/3, 0)$$

$$(2/11, 1/11)$$

Lastly, $3x+5y=1$; $2x+7y=1$

$$\rightarrow 6x+10y=2$$

$$- 6x+21y=3$$

$$11y=5, y=5/11$$

$$2x+7/11=1/11$$

$$2x=2/11$$

$$J = \begin{bmatrix} \frac{\partial}{\partial x} \left(\frac{x}{3x+5y} \right) & \frac{\partial}{\partial y} \left(\frac{x}{3x+5y} \right) \\ \frac{\partial}{\partial x} \left(\frac{y}{2x+7y} \right) & \frac{\partial}{\partial y} \left(\frac{y}{2x+7y} \right) \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{5y}{(3x+5y)^2} & \frac{-5x}{(3x+5y)^2} \\ \frac{-2y}{(2x+7y)^2} & \frac{2x}{(2x+7y)^2} \end{bmatrix}$$

Check SS: (check all 3)

$$a. J(0, 1/7) = \begin{pmatrix} 5/7 & 0 \\ (5/7)^2 & -2/7 \\ -2/7 & 0 \end{pmatrix} = \begin{pmatrix} 7/5 & 0 \\ -2/7 & 0 \end{pmatrix}$$

$$(7/5 - \lambda)(-\lambda) = \lambda^2 - 7/5\lambda = 0 \Rightarrow \lambda(\lambda - 7/5) = 0$$

$$\lambda = 0, \lambda_2 = 7/5 : |\lambda_2| > 1 \rightarrow \text{unstable}$$

$$b. J(1/3, 0) = \begin{pmatrix} 0 & -5/3 \\ 0 & 3/2 \end{pmatrix} \rightarrow (-\lambda)(3/2 - \lambda) = 0$$

$$\text{or } \lambda(\lambda - 3/2) = 0, \lambda_1 = 0, \lambda_2 = 3/2$$

$$\rightarrow |\lambda_2| > 1 \rightarrow \text{unstable}$$

$$c. (2/11, 1/11) : \begin{pmatrix} \frac{5/11}{(6/11+5/11)^2} & \frac{-10/11}{(6/11+5/11)^2} \\ \frac{-2/11}{(4/11+7/11)^2} & \frac{4/11}{(4/11+7/11)^2} \end{pmatrix} = \begin{pmatrix} 5/11 & -10/11 \\ -2/11 & 4/11 \end{pmatrix}$$

$$(5/11 - \lambda)(4/11 - \lambda) - 20/11 = 0 \rightarrow \lambda^2 - 9/11\lambda - 200/121 = 0$$

$$|\lambda_2| > 1 \rightarrow \text{unstable} \quad [SO, \text{no SSS}]$$