1. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3 + a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 + a_1(n)}$$

Which of them is a stable steady-state?

a) $X_1 = \frac{X_1}{3+X_2}$ $\quad X_2 = \frac{X_2}{2+X_1}$

$0 = X_1 - \frac{X_1}{3+X_2}$ $\quad 0 = X_2 - \frac{X_2}{2+X_1}$

$0 = X_1\left(1 - \frac{X_1}{3+X_2}\right)$ $\quad 0 = X_2\left(1 - \frac{X_2}{2+X_1}\right)$

$-1 + \frac{1}{3+X_2} = 0$ $\quad X_2 = -2$ $\quad -2 = \frac{-2}{2+X_1}$

$\frac{1}{3+X_2} = 1$ $\quad -4 - 2X_1 = -2$

$1 = 3 + X_2$ $\quad -2X_1 = 2$

$X_2 = -2$ $\quad X_1 = -1$

$(-1, -2)$

(0,0)

$X_1 = 0 : 1 - \frac{1}{3+X_2} = 0$

$3 + X_2 = 1$

$X_2 = -2$ $\quad (0, -2)$

$X_2 = 0 : 1 - \frac{1}{2+X_1} = 0$

$2 + X_1 = 1$

$X_1 = -1$ $\quad (-1, 0)$

$$J = \begin{bmatrix} \frac{1}{3+X_2} & -\frac{X_1}{(3+X_2)^2} \\ -\frac{X_2}{(2+X_1)^2} & \frac{1}{2+X_1} \end{bmatrix}$$

$$J(-1,-2) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$(1-\lambda)(1-\lambda) - 2 = 0$

$1 - \lambda - \lambda + \lambda^2 - 2 = 0$

$\lambda^2 - 2\lambda - 1 = 0 \longrightarrow \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$: unstable

$$J(0,0) = \begin{bmatrix} \frac{1}{3} & \frac{1}{9} \\ 0 & 1 \end{bmatrix} = \left(\frac{1}{3} - \lambda\right)(1-\lambda) - 0 = 0$$

$\frac{1}{3} - \frac{1}{3}\lambda - \lambda + \lambda^2 = 0$

$\lambda^2 - \frac{4}{3}\lambda + \frac{1}{3} = 0$

$(\lambda - 1)(\lambda - \frac{1}{3})$

$\lambda = 1 \therefore$ unstable

$$J(0,-2) \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$(\lambda - 1)(\lambda - \frac{1}{2}) = 0$

$\lambda^2 - \frac{3}{2}\lambda + \frac{1}{2} = 0$

$(\lambda - 1)(\lambda - \frac{1}{2})$ unstable

2. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3a_1(n) + 5a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2a_1(n) + 7a_2(n)}$$

Which of them is a stable steady-state?

$X_1 = \frac{X_1}{3X_1 + 5X_2}$ $\qquad$ $X_2 = \frac{X_2}{2X_1 + 7X_2}$

$X_1 - \frac{X_1}{3X_1 + 5X_2} = 0$ $\qquad$ $X_2 - \frac{X_2}{2X_1 + 7X_2} = 0$

$X_2 = 0 : X_1 - \frac{X_1}{3X_1} = 0$ $\qquad$ $X_1 = 0 : X_2 - \frac{X_2}{7X_2} = 0$

$X_1 = \frac{1}{3} \rightarrow (\frac{1}{3}, 0)$ $\qquad$ $X_2 = \frac{1}{7} \rightarrow (0, \frac{1}{7})$

$(3X_1 + 5X_2 = 1)\,2$

$(2X_1 + 7X_2 = 1)\,3$

$6X_1 + 10X_2 = 2$

$-\,6X_1 + 21X_2 = 3$

$-11X_2 = -1$

$X_2 = \frac{1}{11}$

$3X_1 + 5(\frac{1}{11}) = 1$

$3X_1 + \frac{5}{11} = 1$

$3X_1 = \frac{11}{11} - \frac{5}{11}$

$3X_1 = \frac{6}{11}$

$X_1 = \frac{2}{11}$

$(\frac{2}{11}, \frac{1}{11})$

$$J = \begin{bmatrix} \frac{5X_2}{(3X_1 + 5X_2)^2} & \frac{-5X_1}{(3X_1 + 5X_2)^2} \\ \frac{-2X_2}{(2X_1 + 7X_2)^2} & \frac{2X_1}{(2X_1 + 7X_2)^2} \end{bmatrix}$$

$J(\frac{1}{3}, 0) = \begin{bmatrix} 0 & -5/3 \\ 0 & 2/3 \end{bmatrix}$ $\qquad$ $(\lambda)(\lambda - \frac{2}{3}) = 0$

$\lambda^2 - \frac{2}{3}\lambda = 0$

$\lambda = 0, 2/3 \therefore$ stable

$J(0, \frac{1}{7}) = \begin{bmatrix} 5/7 & 0 \\ -2/7 & 0 \end{bmatrix}$ $\qquad$ $(\lambda - \frac{5}{7})(\lambda) = 0$

$\lambda = 0, 5/7 \therefore$ stable

$J(\frac{2}{11}, \frac{1}{11}) = \begin{bmatrix} 5/11 & -10/11 \\ -2/11 & 4/11 \end{bmatrix}$ $\qquad$ $(\lambda - \frac{5}{11})(\lambda - \frac{4}{11}) - \frac{20}{121} = 0$

$\lambda^2 - \frac{9}{11}\lambda + \frac{20}{124} - \frac{20}{124} = 0$

$\lambda^2 - \frac{9}{11}\lambda = 0$

$\lambda = 0, 9/11$ stable