HW13

① $x_1 = \frac{x_1}{3+x_2}$ $\quad x_2 = \frac{x_2}{2+x_1}$

$x_1\left(1 - \frac{1}{3+x_2}\right) = 0$ $\quad x_2\left(1 - \frac{1}{2+x_1}\right) = 0$

$x_1 = 0$ OR $\quad x_2 = 0$ OR

$x(3+x)^{-1}$ $\quad 3+x_2 = 1 \rightsquigarrow x_2 = -2$ $\quad 2+x_1 = 1 \rightsquigarrow x_1 = -1$

$(0,0), (0,-2), (-1,0)$

$J = \begin{bmatrix} \frac{1}{3+x_2} & \frac{-x_1}{(x_2+3)^2} \\ \frac{-x_2}{(x_1+2)^2} & \frac{1}{2+x_1} \end{bmatrix}$ $\quad \boxed{J(0,0)} = \begin{bmatrix} 1/3 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ $(1/3 - \lambda)(\frac{1}{2} - \lambda)$

$1/3 < 0, \frac{1}{2} < 1$ $\quad \boxed{\therefore \text{STABLE}}$

$J(0,-2) = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ $(1-\lambda)(\frac{1}{2}-\lambda) = 0$ $\quad |\lambda| = 1, \frac{1}{2}$

SEMI-STABLE

$J(-1,0) = \begin{bmatrix} \frac{1}{3} & \frac{1}{9} \\ 0 & 1 \end{bmatrix}$ $(\frac{1}{3} - \lambda)(1-\lambda) = 0$

SEMISTABLE

② $x\left(1 - \frac{1}{3x+5y}\right) = 0$; $x = 0$ $\quad \begin{cases} 3x+5y = 1 \\ 2x+7y = 1 \end{cases}$ $(2/11, 1/11)$

$\frac{x}{3x+5y}$ $\quad y\left(1 - \frac{1}{2x+7y}\right) = 0$; $y = 0$

$(0, \frac{1}{5}), (\frac{1}{2}, 0)$

$\frac{-2}{5} \cdot \frac{25}{4}$

$J = \begin{bmatrix} \frac{5y}{(3x+5y)^2} & \frac{-5x}{(3x+5y)^2} \\ \frac{-2y}{(2x+7y)^2} & \frac{2x}{(2x+7y)^2} \end{bmatrix}$ $\quad J(0, 1/5) = \begin{bmatrix} 1 & 0 \\ -\frac{5}{2} & 0 \end{bmatrix}$ $-\lambda(1-\lambda) = 0$

semistable

$\frac{2}{5} \cdot \frac{25}{4}$

$J(1/5, 0) = \begin{bmatrix} 0 & -1 \\ 0 & 5/2 \end{bmatrix}$ $\quad J(2/11, 1/11) = \begin{bmatrix} .604 & -.909 \\ -.1818 & .3636 \end{bmatrix}$

$-\lambda(5/2 - \lambda) = 0$ UNSTABLE $\quad (.604-\lambda)(.3636-\lambda) - .1645 = 0$

STABLE