

## Homework for Lecture 13 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 20, 2025.

Subject: hw13

with an attachment hw13FirstLast.pdf

1. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3 + a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 + a_1(n)}$$

$$x = \frac{x}{3+y} \Rightarrow x(2+y) = 0 \rightarrow x=0, y=-2$$

$$y = \frac{y}{2+x} \Rightarrow y(1+x) = 0 \rightarrow y=0, x=-1$$

$$(0,0) \quad (-1,-2)$$

$$J = \begin{pmatrix} \frac{1}{3+y} & -\frac{x}{(3+y)^2} \\ \frac{y}{(2+x)^2} & \frac{1}{2+x} \end{pmatrix}$$

$$J(0,0) = \begin{pmatrix} 1/3 & 0 \\ 0 & 1/2 \end{pmatrix} \rightarrow \text{stable}$$

$$J(-1,-2) = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \rightarrow \text{unstable}$$

Which of them is a stable steady-state?

2. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3a_1(n) + 5a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2a_1(n) + 7a_2(n)}$$

$$x = \frac{x}{3x+5y} \rightarrow x=0, 3x+5y=1$$

$$x=0 \rightarrow 3x+5y=1 \rightarrow y=1/5 \rightarrow (0, 1/5)$$

$$y = \frac{y}{2x+7y} \rightarrow y=0, 2x+7y=1$$

$$y=0 \rightarrow 2x+7y=1 \rightarrow x=1/2 \rightarrow (1/2, 0)$$

$$3x+5y=1, 2x+7y=1 \rightarrow x=2/11, y=1/11$$

$$J = \begin{pmatrix} \frac{1}{3x+5y} - \frac{3x}{(3x+5y)^2} & -\frac{5x}{(3x+5y)^2} \\ -\frac{2y}{(2x+7y)^2} & \frac{1}{2x+7y} - \frac{7y}{(2x+7y)^2} \end{pmatrix}$$

Which of them is a stable steady-state?

$$J(1/3, 0) = \begin{pmatrix} 0 & -5/3 \\ 0 & 3/2 \end{pmatrix}$$

unstable

$$J(0, 1/5) = \begin{pmatrix} 7/5 & 0 \\ -2/7 & 0 \end{pmatrix}$$

unstable

$$J(2/11, 1/11) = \begin{pmatrix} 5/11 & -10/11 \\ -2/11 & 4/11 \end{pmatrix}$$

Stable