Homework for Lecture 13 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 20, 2025.

Subject: hw13

with an attachment hw13FirstLast.pdf

1. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3 + a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 + a_1(n)}$$

Which of them is a stable steady-state?

2. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3a_1(n) + 5a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2a_1(n) + 7a_2(n)}$$

Which of them is a stable steady-state?

1. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3 + a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 + a_1(n)}$$

Which of them is a stable steady-state?

$x_1 = \frac{x_1}{3+x_2}$ $\quad x_2 = \frac{x_2}{2+x_1}$

$x_1 - \frac{x_1}{3+x_2} = 0$

$x_1\left(1 - \frac{1}{3+x_2}\right) = 0$

$-1 + \frac{1}{3+x_2} = 0$

$\frac{1}{3+x_2} = 1$

$1 = 3 + x_2$

$-2 = x_2$

$x_2 = -2: \quad -2 = \frac{-2}{2+x_1}$

$-4 - 2x_1 = -2$

$-2x_1 = 2$

$x_1 = -1$

$(-1,-2)$

$(0,0)$

$x_1 = 0: \quad 1 - \frac{1}{3+x_2} = 0$

$3 + x_2 = 1$

$x_2 = -2 \quad (0,-2)$

$x_2 = 0: \quad 1 - \frac{1}{2+x_1} = 0$

$2 + x_1 = 1$

$x_1 = -1 \quad (-1,0)$

$$J = \begin{bmatrix} \frac{1}{3+x_2} & \frac{-x_1}{(3+x_2)^2} \\ \frac{-x_2}{(2+x_1)^2} & \frac{1}{2+x_1} \end{bmatrix}$$

$J(-1,-2) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ $\quad (\lambda-1)(\lambda-1) - 2 = 0$

$\lambda^2 - 2\lambda + 1 - 2 = 0$

$\lambda^2 - 2\lambda - 1 = 0$

$\frac{2 \pm \sqrt{4+4}}{2}$

$\frac{2 \pm 2\sqrt{2}}{2}$

$1 \pm \sqrt{2}$ ∴ unstable

$J(0,0) \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ $\quad (\lambda - \frac{1}{3})(\lambda - \frac{1}{2}) = 0$

$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0$

$(\lambda - \frac{1}{3})(\lambda - \frac{1}{2})$ both <1 ∴ stable

$J(-1,0) \begin{bmatrix} \frac{1}{3} & \frac{1}{9} \\ 0 & 1 \end{bmatrix}$ $\quad (\lambda - \frac{1}{3})(\lambda - 1) = 0$

$\lambda^2 - \frac{4}{3}\lambda + \frac{1}{3} = 0$

$(\lambda - 1)(\lambda - \frac{1}{3})$ $\quad \lambda = 1$ ∴ unstable

$J(0,-2) \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ $\quad (\lambda - 1)(\lambda - \frac{1}{2}) = 0$

$\lambda^2 - \frac{3}{2}\lambda + \frac{1}{2} = 0$

$(\lambda - 1)(\lambda - \frac{1}{2})$ unstable

2. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3a_1(n) + 5a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2a_1(n) + 7a_2(n)}$$

Which of them is a stable steady-state?

$x_1 = \frac{x_1}{3x_1+5x_2}$ $\qquad$ $x_2 = \frac{x_2}{2x_1+7x_2}$

$x_1 - \frac{x_1}{3x_1+5x_2} = 0$ $\qquad$ $x_2 - \frac{x_2}{2x_1+7x_2} = 0$

$x_2=0$: $x_1 - \frac{\cancel{x_1}}{3\cancel{x_1}} = 0$ $\qquad$ $x_1=0$: $x_2 - \frac{\cancel{x_2}}{7\cancel{x_2}} = 0$

$x_1 = \frac{1}{3}$ $\qquad$ $x_2 = \frac{1}{7}$

$(\frac{1}{3}, 0)$ $\qquad$ $(0, \frac{1}{7})$

$3x_1+5x_2 = 1$ $\qquad$ $2x_1+7x_2=1$

$x_1 = \frac{1}{2} - \frac{7}{2}x_2$

$\frac{3}{2} - \frac{21}{2}x_2 + 5x_2 = 1$

$-\frac{11}{2}x_2 = -\frac{1}{2}$

$x_2 = \frac{1}{11}$

$x_1 = \frac{4}{22} = \frac{2}{11}$

$(\frac{2}{11}, \frac{1}{11})$

$$J = \begin{bmatrix} \frac{5x_2}{(3x_1+5x_2)^2} & \frac{-5x_1}{(3x_1+5x_2)^2} \\ \frac{-2x_2}{(2x_1+7x_2)^2} & \frac{2x_1}{(2x_1+7x_2)^2} \end{bmatrix}$$

$$J(\tfrac{1}{3},0) = \begin{bmatrix} 0 & -5/3 \\ 0 & 2/3 \end{bmatrix}$$

$(\lambda)(\lambda - \frac{2}{3}) = 0$

$\lambda^2 - \frac{2}{3}\lambda = 0$

$\lambda = 0, \frac{2}{3}$ $\qquad$ $\therefore$ stable

$$J(0,\tfrac{1}{7}) = \begin{bmatrix} \frac{5}{7} & 0 \\ -\frac{2}{7} & 0 \end{bmatrix}$$

$(\lambda - \frac{5}{7})(\lambda) = 0$

$\lambda = 0, \frac{5}{7}$ $\qquad$ $\therefore$ stable

$$J(\tfrac{2}{11},\tfrac{1}{11}) \begin{bmatrix} 5/11 & -10/11 \\ -2/11 & 4/11 \end{bmatrix}$$

$(\lambda - \frac{5}{11})(\lambda - \frac{4}{11}) - \frac{20}{121} = 0$

$\lambda^2 - \frac{9}{11}\lambda + \cancel{\frac{20}{121}} - \cancel{\frac{20}{121}} = 0$

$\lambda^2 - \frac{9}{11}\lambda = 0$

$\lambda = 0, \frac{9}{11}$ $\qquad$ Stable