Homework for Lecture 13 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 20, 2025.

Subject: hw13

with an attachment hw13FirstLast.pdf

1. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3 + a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 + a_1(n)}$$

b Which of them is a stable steady-state?

2. a Find all the steady-states of the system

$$a_1(n+1) = \frac{a_1(n)}{3a_1(n) + 5a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2a_1(n) + 7a_2(n)}$$

b Which of them is a stable steady-state?

Daniyal Chaudhry HW 13

1. a) $\left.\begin{array}{l} q_1(n+1) = q_1(n)/(3+q_2(n)) \\ q_2(n+1) = q_2(n)/(2+q_1(n)) \end{array}\right\}$ $(x_1, x_2) \to (x_1/(3+x_2),\ x_2/(2+x_1))$

$$x_1 - \frac{x_1}{(3+x_2)} = 0 \to x_1\left(1 - \frac{1}{3+x_2}\right) = 0$$

$$x_2 - \frac{x_2}{(2+x_1)} = 0 \to x_2\left(1 - \frac{1}{2+x_1}\right) = 0$$

Steady States: $(0,0)$, $(-1,-2)$

(0,0) works as a steadystate. in fact, if x_1 is 0, then x_2 must be 0 to achieve a SS, and vice versa. $(-1,-2)$ also works, producing $(1-\frac{1}{1})$ in both equations.

b) J: $\begin{bmatrix} \frac{1}{3+x_2} & \frac{-x_1}{(3+x_2)^2} \\ \frac{-x_2}{(2+x_1)^2} & \frac{1}{2+x_1} \end{bmatrix}$

$J(0,0) = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

$\lambda = 1/2, 1/3$, both <1

(0,0) is stable

$J(-1,-2) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$\lambda^2 - 2\lambda + 1 - 2$

$\lambda = \frac{2}{2} \pm \frac{\sqrt{4+4}}{2} \Rightarrow \lambda_1 = 1+\sqrt{2} \to (-1,-2)$ is unstable

2) a) $X_1 = \frac{x_1}{3x_1+5x_2}$ $\quad X_2 = \frac{x_2}{2x_1+7x_2}$: $\quad x_1\left(1 - \frac{1}{3x_1+5x_2}\right) = 0$, $\quad x_2\left(1 - \frac{1}{2x_1+7x_2}\right) = 0$

(0,0) doesn't work actually neither can be 0 in this system.

(denominators = 1)

$$\begin{array}{r} 3x_1 + 5x_2 = 1 \\ -(2x_1 + 7x_2 = 1) \\ \hline x_1 - 2x_2 = 0 : x_1 = 2x_2 \end{array}$$

$11x_2 = 1$ $\quad x_2 = \frac{1}{11}$, $\quad x_1 = \frac{2}{11}$

Only SS: $\left(\frac{2}{11}, \frac{1}{11}\right)$

b) $\frac{\partial f_1}{\partial x_1}$: $\frac{(3x_1+5x_2)(1) - (x_1)(3)}{\text{denom}^2} = \frac{5x_2}{\text{denom}^2}$

$\frac{\partial f_1}{\partial x_2}$: $\frac{(3x_1+5x_2)(0) - (x_1)(5)}{\text{denom}^2} = \frac{-5x_1}{\text{denom}^2}$

same process for f_2:

$J = \begin{bmatrix} \frac{5x_2}{(3x_1+5x_2)^2} & \frac{-5x_1}{(3x_1+5x_2)^2} \\ -\frac{2x_2}{(2x_1+7x_2)^2} & \frac{2x_1}{(2x_1+7x_2)^2} \end{bmatrix}$

$J\left(\frac{2}{11}, \frac{1}{11}\right) = \begin{bmatrix} \frac{5}{11} & \frac{-10}{11} \\ \frac{-2}{11} & \frac{4}{11} \end{bmatrix}$

using Trace/Determinants Method (as it is faster, determinant is 0)

Trace: $\frac{9}{11}$

Determinant: 0 as $(5\times4) - (-2\times-10) = 0$ $\Big\}$ so λ_1 must be 0, $\lambda_2 = \frac{9}{11}$ $\Big\}$ both are <1 $\left(\frac{2}{11}, \frac{1}{11}\right)$ stable