**Homework for Lecture 12 of Dr. Z.'s Dynamical Models in Biology class**

Email the answers ( as .pdf file ) to

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by 8:00pm Monday, Oct. 20, 2025.

Subject: hw12

with an attachment hw8FirstLast.pdf and/or hw12FirstLast.txt

Using

`http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

**1.** What is the ultimate period of

`evalf(Orb((1-x)*(2-x)*(3-x)/12,x,0.5,1000,1010),10);`?

By Playing 'high-low', estimate the cut-offs $r_1$, $r_2$, $r_3$, $r_4$ such that

the recurrence

$$a(n+1) = (1-a(n))(2-a(n))(3-a(n))/r \quad ,$$

has

L:=Zeig( ) ?

[3, 3.449489 7, 3.5440903, 3.5644073.
3.5687594, 3.5696916, 3.5698913, 3.5699340]

- ultimate period 1 for $r_1 < r$
- ultimate period 2 for $r_2 < r < r_1$
- ultimate period 4 for $r_3 < r < r_2$

(challenging!):

- ultimate period 8 for $r_4 < r < r_3$

**2.**

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

```
T:=RT([x,y],30); SSg(T,[x,y]); ORB(T,[x,y,z],[100., 200.],1000,1020);
```

Here the first command, `T:=RT([x,y],30);` generates a random rational transformation in $R^2$, the second command, SSg(T,[x,y]);, finds steady-states (i.e. fixed points of the transformation `T`, using

Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y],[100.,200.],1000,1020);` finds the long-term behavior. [1.702668856, 2.883800265]

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

`ORB(T,[x,y],[100., 200.],1000,1020)[-1];` [1.702668856, 2.883800265]

was not part of the output of

SSg(T,[x,y]); [1.702668856, 2.883800265]

**3.**

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

`T:=RT([x,y,z],10); SSg(T,[x,y,z]); ORB(T,[x,y,z],[100., 200.,300.],1000,1020);`

Here the first command, `T:=RT([x,y,z],10);` generates a random rational transformation in $R^3$, the second command, SSg(T,[x,y,z]);, finds steady-states (i.e. fixed points of the transformation `T`, using Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y,z],[100., 200.,300.],1000,1020);` finds the long-term behavior.

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

`ORB(T,[x,y,z],[100., 200.,300.],1000,1020)[-1];`

was not part of the output of [0.836291612, 1.289106652, 0.681552806]

SSg(T,[x,y,z]); [0.836291612, 1.289106652, 0.681552806].

same!

```
> read `DMB12.txt`;
`For a list of the Main procedures type: Help12(); for help with a
specific pro\
cedure type: Help12(ProcedureName); for example Help12(Feig);`

> evalf(Orb((1-x)*(2-x)*(3-x)/12,x,0.5,1000,1010),10);
[.2804250993, .2804250993, .2804250993, .2804250993, .2804250993,
.2804250993,
.2804250993, .2804250993, .2804250993, .2804250993, .2804250993]

> Help12();
`The available procedures are: ChaosStory, Feig, Orb, ORB, Orbk,
RecToT, SS, SS\
g, SSS, RT `

> Help12(RT);
`RT(var,K): A random rational transformation of numerator and
denominator degre\
es  1 from R^k to R^k (where k=nops(var), with postiive integer
coefficients fr\
om 1 to K The inpus are a list of variables x and a posi. integer K`
`is for generating examples`
`Try:`
`RT([x,y],10);`

> Help12(Feig);
`Feig(): The list (from wikipedia) of length 8, of the period-
doubling cut-offs\
`
`of the logistic equation x(n+1)=r*x(n)*(1-x(n)). Calling this list
L, i.e.`
`L:=Feig(): The r below L[1] every orbit tends to ONE ultimate value`
`for r between L[1] and L[2], in the long run it alternates between 2
ultimate \
values`
`between L[2] and L[3] , in the long run it alternates between 4=2^2
ultimate v\
alues`
`Between L[i] and L[i+1], in the long run it alternates between 2^i
ultimate va\
lues. Try:`
`Feig():`
```

```
> L:=Feig()
Typesetting:-mprintslash([(L := [3, 3.4494897, 3.5440903, 3.5644073,
3.5687594,
3.5696916, 3.5698913, 3.5699340])],[[3, 3.4494897, 3.5440903,
3.5644073, 3.5687\
594, 3.5696916, 3.5698913, 3.5699340]])

> T:=RT([x,y],30);
Typesetting:-mprintslash([(T := [(29+23*x+15*y)/(26+13*x+6*y), (2+27*
x+12*y)/(4
+6*x+5*y)])],[[(29+23*x+15*y)/(26+13*x+6*y), (2+27*x+12*y)/(4+6*x+5*
y)]])

> SSg(T,[x,y]);
[[1.702668856, 2.883800265]]

> ORB(T,[x,y,z],[100.,200.],1000,1020);
`bad input`
FAIL

> ORB(T,[x,y],[100.,200.],1000,1020);
[[1.702668856, 2.883800265], [1.702668856, 2.883800265],
[1.702668856, 2.883800\
265], [1.702668856, 2.883800265], [1.702668856, 2.883800265],
[1.702668856, 2.8\
83800265], [1.702668856, 2.883800265], [1.702668856, 2.883800265],
[1.702668856
, 2.883800265], [1.702668856, 2.883800265], [1.702668856,
2.883800265], [1.7026\
68856, 2.883800265], [1.702668856, 2.883800265], [1.702668856,
2.883800265], [1\
.702668856, 2.883800265], [1.702668856, 2.883800265], [1.702668856,
2.883800265
], [1.702668856, 2.883800265], [1.702668856, 2.883800265],
[1.702668856, 2.8838\
00265], [1.702668856, 2.883800265]]

> ORB(T,[x,y],[100.,200.],1000,1020)[-1];

[1.702668856, 2.883800265]

> SSg(T,[x,y]);
[[1.702668856, 2.883800265]]
```

```
> T:=RT([x,y,z],10);
Typesetting:-mprintslash([(T := [(5+10*x+2*y+2*z)/(4+8*x+3*y+9*z),
(10+2*x+8*y+
10*z)/(9+x+6*y+7*z), (7+3*x+3*y+5*z)/(3+8*x+10*y+3*z)])],[[(5+10*x+2*
y+2*z)/(4+
8*x+3*y+9*z), (10+2*x+8*y+10*z)/(9+x+6*y+7*z), (7+3*x+3*y+5*z)/(3+8*
x+10*y+3*z)
]])

> SSg(T,[x,y,z]);
[[.83628902, 1.289106652, .68155329]]

> ORB(T,[x,y,z],[100.,200.,300.],1000,1020);
[[.8362916127, 1.289106652, .6815528061], [.8362916127, 1.289106652,
.681552806\
1], [.8362916127, 1.289106652, .6815528061], [.8362916127,
1.289106652, .681552\
8061], [.8362916127, 1.289106652, .6815528061], [.8362916127,
1.289106652, .681\
5528061], [.8362916127, 1.289106652, .6815528061], [.8362916127,
1.289106652, .\
6815528061], [.8362916127, 1.289106652, .6815528061], [.8362916127,
1.289106652
, .6815528061], [.8362916127, 1.289106652, .6815528061],
[.8362916127, 1.289106\
652, .6815528061], [.8362916127, 1.289106652, .6815528061],
[.8362916127, 1.289\
106652, .6815528061], [.8362916127, 1.289106652, .6815528061],
[.8362916127, 1.\
289106652, .6815528061], [.8362916127, 1.289106652, .6815528061],
[.8362916127,
1.289106652, .6815528061], [.8362916127, 1.289106652, .6815528061],
[.836291612\
7, 1.289106652, .6815528061], [.8362916127, 1.289106652, .6815528061]
]

> ORB(T,[x,y,z],[100.,200.,300.],1000,1020)[-1];
[.8362916127, 1.289106652, .6815528061]

> SSg(T,[x,y,z]);
[[.83628902, 1.289106652, .68155329]]
```