Homework for Lecture 12 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Oct. 20, 2025.

Subject: hw12

with an attachment hw8FirstLast.pdf and/or hw12FirstLast.txt

Using

`http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

1. What is the ultimate period of

`evalf(Orb((1-x)*(2-x)*(3-x)/12,x,0.5,1000,1010),10);`?

By Playing 'high-low', estimate the cut-offs r_1, r_2, r_3, r_4 such that

the recurrence

$$a(n+1) = (1-a(n))(2-a(n))(3-a(n))/r \quad ,$$

has

• ultimate period 1 for $r_1 < r$

• ultimate period 2 for $r_2 < r < r_1$

• ultimate period 4 for $r_3 < r < r_2$

(challenging!):

• ultimate period 8 for $r_4 < r < r_3$

2.

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

```
T:=RT([x,y],30); SSg(T,[x,y]); ORB(T,[x,y,z],[100., 200.],1000,1020);
```

Here the first command, `T:=RT([x,y],30);` generates a random rational transformation in R^2, the second command, SSg(T,[x,y]);, finds steady-states (i.e. fixed points of the transformation `T`, using

Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y],[100., 200.],1000,1020);` finds the long-term behavior.

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

`ORB(T,[x,y],[100., 200.],1000,1020)[-1];`

was not part of the output of

SSg(T,[x,y]);

3.

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

`T:=RT([x,y,z],10); SSg(T,[x,y,z]); ORB(T,[x,y,z],[100., 200.,300.],1000,1020);`

Here the first command, `T:=RT([x,y,z],10);` generates a random rational transformation in R^3, the second command, SSg(T,[x,y,z]);, finds steady-states (i.e. fixed points of the transformation `T`, using Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y,z],[100., 200.,300.],1000,1020);` finds the long-term behavior.

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

`ORB(T,[x,y,z],[100., 200.,300.],1000,1020)[-1];`

was not part of the output of

SSg(T,[x,y,z]);

1. What is the ultimate period of

```
evalf(Orb((1-x)*(2-x)*(3-x)/12,x,0.5,1000,1010),10);?
```

period = 1

```
> evalf( Orb( (1-x)·(2-x)·(3-x)/12, x, .5, 1000, 1010 ), 10 );
[0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993,
 0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993]
```

By Playing 'high-low', estimate the cut-offs r_1, r_2, r_3, r_4 such that the recurrence

$$a(n+1) = (1-a(n))(2-a(n))(3-a(n))/r$$

has

- ultimate period 1 for $r_1 < r$ $r > 9$
- ultimate period 2 for $r_2 < r < r_1$ $9 < r < 7.8$
- ultimate period 4 for $r_3 < r < r_2$ $7.8 < r < 7.4$

(challenging!):

- ultimate period 8 for $r_4 < r < r_3$ $7.4 < r < 7.3$

2.

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

```
T:=RT([x,y],30); SSg(T,[x,y]); ORB(T,[x,y,z],[100., 200.],1000,1020);
```

Here the first command, `T:=RT([x,y],30);` generates a random rational transformation in R^2, the second command, SSg(T,[x,y]);, finds steady-states (i.e. fixed points of the transformation `T`, using Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y],[100., 200.],1000,1020);` finds the long-term behavior.

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

`ORB(T,[x,y],[100., 200.],1000,1020)[-1];`

was not part of the output of

SSg(T,[x,y]);

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{14 + 30x + 20y}{16 + 29x + 19y}, \frac{3 + 9x + 13y}{3 + 8x + 7y}\right]$$
$SS := [[-0.6268219224, 0.1933515201]]$
$OR := [1.006383440, 1.459009550]$

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{9 + 10x + 17y}{28 + 5x + 25y}, \frac{1 + 30x + y}{19 + 9x + 9y}\right]$$
$SS := [[0.5385368928, 0.6061718956]]$
$OR := [0.5385368928, 0.6061718956]$

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{27 + 11x + 25y}{20 + 30x + 14y}, \frac{13 + 12x + 9y}{18 + 21x + 5y}\right]$$
$SS := [[-1.145939322, 0.05072354173]]$
$OR := [0.9514143582, 0.7464262948]$

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{24 + 17x + 18y}{9 + 5x + 12y}, \frac{8 + 28x + 29y}{28 + 2x + y}\right]$$
$SS := [[1.788689382, 6.440736406]]$
$OR := [1.788689382, 6.440736406]$

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{12 + 9x + 30y}{14 + 13x + 15y}, \frac{23 + 17x + 9y}{21 + 14x + 17y}\right]$$
$SS := [[1.176137695, 0.9599975484]]$
$OR := [1.176137695, 0.9599975484]$

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{28 + 29x + 20y}{15 + 24x + 5y}, \frac{17 + 21x + 10y}{21 + 5x + 5y}\right]$$
$SS := [[1.747913138, 1.851973346]]$
$OR := [1.747913138, 1.851973346]$

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{26 + 3x + 22y}{22 + 23x + y}, \frac{4 + 28x + 14y}{18 + 24x + 2y}\right]$$
$SS := [[1.094536174, 1.069117841]]$
$OR := [1.094536174, 1.069117840]$

$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{18 + 4x + 17y}{29 + 18x + 21y}, \frac{22 + 10x + 5y}{21 + x + 13y}\right]$$
$SS := [[-2.044168692, 0.1019738841]]$
$OR := [0.6113881224, 0.9644715537]$ (9)

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{2 + 8x + 26y}{2 + 5x + 29y}, \frac{1 + 29x + 3y}{25 + 29x + 2y}\right]$$
$SS := [[0.8917986486, -24.47974154]]$
$OR := [1.057868270, 0.5882166446]$ (10)

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{2 + 12x + 28y}{19 + 21x + 4y}, \frac{4 + 21x + 27y}{8 + 12x + 11y}\right]$$
$SS := [[1.293602558, 1.848660975]]$
$OR := [1.293602558, 1.848660976]$ (11)

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{28 + 7x + 17y}{27 + 18x + 14y}, \frac{20 + 18x + 18y}{19 + 12x + 2y}\right]$$
$SS := [[-2.775900883, 0.9878386067]]$
$OR := [0.9606053208, 2.202386800]$ (12)

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{22 + 20x + 14y}{10 + 20x + 4y}, \frac{13 + 23x + 13y}{5 + 28x + y}\right]$$
$SS := [[0.03670629272, -1.612635093]]$
$OR := [1.546388098, 1.326062000]$ (13)

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

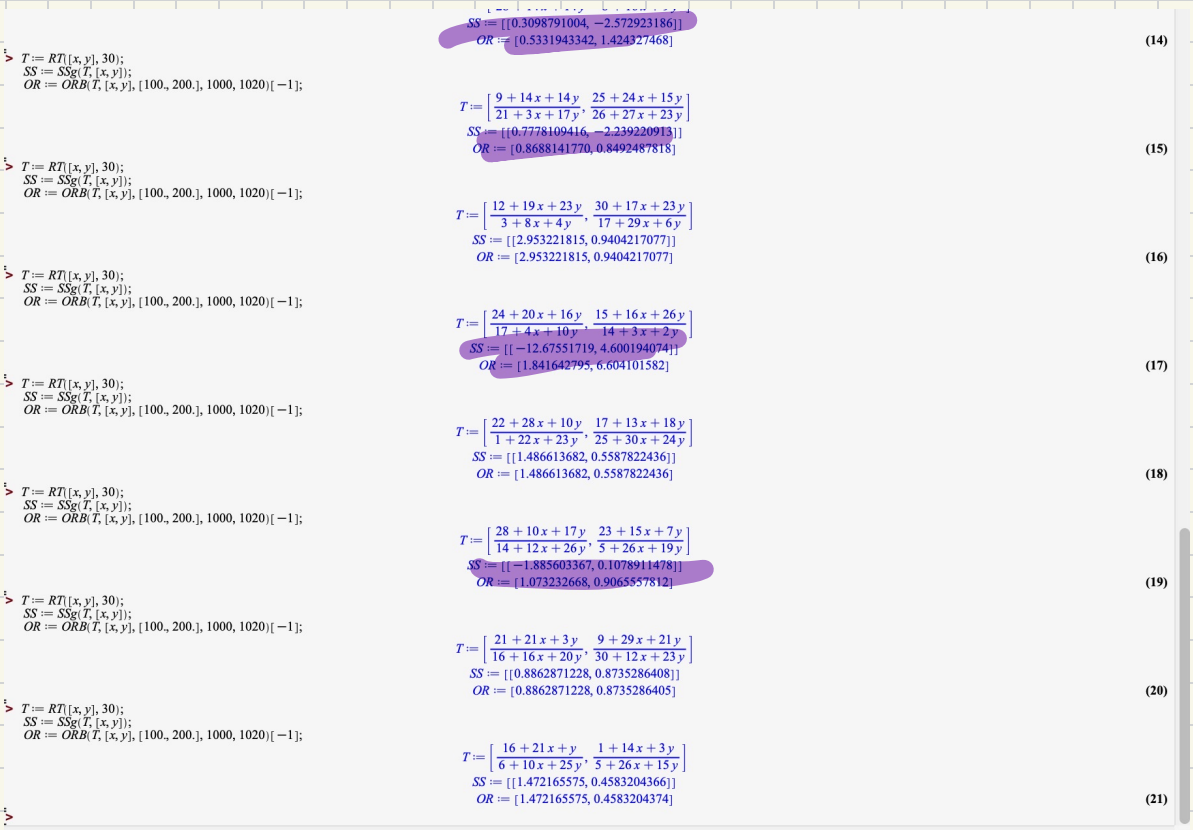
$$T := \left[\frac{5 + 30x + 6y}{28 + 14x + 14y}, \frac{29 + 16x + 3y}{8 + 16x + 9y}\right]$$
$SS := [[0.3098791004, -2.572923186]]$
$OR := [0.5331943342, 1.424327468]$ (14)

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{9 + 14x + 14y}{21 + 3x + 17y}, \frac{25 + 24x + 15y}{26 + 27x + 23y}\right]$$
$SS := [[0.7778109416, -2.239220913]]$
$OR := [0.8688141770, 0.8492487818]$ (15)

$T := RT([x, y], 30);$
$SS := SSg(T, [x, y]);$
$OR := ORB(T, [x, y], [100., 200.], 1000, 1020)[-1];$

$$T := \left[\frac{12 + 19x + 23y}{3 + 8x + 4y}, \frac{30 + 17x + 23y}{17 + 29x + 6y}\right]$$



3.

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

```
T:=RT([x,y,z],10); SSg(T,[x,y,z]); ORB(T,[x,y,z],[100., 200.,300.],1000,1020);
```

Here the first command, `T:=RT([x,y,z],10);` generates a random rational transformation in R^3, the second command, SSg(T,[x,y,z]);, finds steady-states (i.e. fixed points of the transformation `T`, using Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y,z],[100., 200.,300.],1000,1020);` finds the long-term behavior.

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

```
ORB(T,[x,y,z],[100., 200.,300.],1000,1020)[-1];
```

was not part of the output of

SSg(T,[x,y,z]);

```
T := RT[{x, y, z}, 10];
SS := SSg[T, {x, y, z}];
OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{7+7x+3y+3z}{5+3x+8y+10z}, \frac{3+x+y+3z}{5+5x+10y+10z}, \frac{10+2x+8y+4z}{2+9x+5y+9z}\right\}$$

$$SS := \{\{0.86972449, 0.319641621, 0.9136189277\}\}$$

$$OR := \{0.8697244783, 0.3196416234, 0.9136189277\}$$

```
T := RT[{x, y, z}, 10];
SS := SSg[T, {x, y, z}];
OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{9+10x+4y+9z}{7+8x+9y+z}, \frac{10+2x+4y+7z}{8+x+6y+4z}, \frac{1+7x+2y+9z}{5+9x+9y+4z}\right\}$$

$$SS := \{\{0.1258080091, 0.5909267246, -1.233399036\}\}$$

$$OR := \{1.122213111, 1.142669001, 0.5938782145\}$$

```
T := RT[{x, y, z}, 10];
SS := SSg[T, {x, y, z}];
OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{3+4x+6y+4z}{4+10x+8y+2z}, \frac{7+2x+5y+3z}{5+7x+5y+4z}, \frac{6+8x+10y+2z}{6+8x+4y+8z}\right\}$$

$$SS := \{\{0.2358593934, 1.806113496, -2.798097198\}\}$$

$$OR := \{0.7415466947, 0.8540037777, 0.9699055803\}$$

```
T := RT[{x, y, z}, 10];
SS := SSg[T, {x, y, z}];
OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{7+9x+4y+10z}{3+2x+9y+z}, \frac{1+5x+2y+4z}{4+x+6y+3z}, \frac{1+7x+2y+z}{5+10x+3y+10z}\right\}$$

$$SS := \{\{2.00267350, 1.085021525, 0.5278976602\}\}$$

$$OR := \{2.002673503, 1.085021526, 0.5278976602\}$$

```
T := RT[{x, y, z}, 10];
SS := SSg[T, {x, y, z}];
OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{4+6x+8y+8z}{7+9x+6y+z}, \frac{2+9x+5y+3z}{2+5x+3y+9z}, \frac{8+5x+4y+3z}{3+9x+3y+8z}\right\}$$

$$SS := \{\{0.9672067670, -52.52048760, 17.17119755\}\}$$

$$OR := \{1.085352615, 1.062996498, 0.8830022932\}$$

```
T := RT[{x, y, z}, 10];
SS := SSg[T, {x, y, z}];
OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{7+9x+10y+z}{5+9x+y+z}, \frac{3+9x+9y+9z}{4+9x+2y+5z}, \frac{5+8x+y+2z}{9+5x+8y+2z}\right\}$$

$$SS := \{\{-1.01899018, 0.6547014955, -0.39282986\}\}$$

$$OR := \{1.683487146, 1.472953021, 0.6973365632\}$$

```
T := RT[{x, y, z}, 10];
SS := SSg[T, {x, y, z}];
OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{1+9x+7y+z}{9+5x+y+4z}, \frac{8+5x+y+5z}{10+5x+5y+5z}, \frac{10+3x+6y+6z}{7+x+4y+2z}\right\}$$

$$SS := \{\{0.6764921777, -0.395492315, -2.22108828\}\}$$

$$OR := \{0.7107377216, 0.8079375359, 1.929292066\}$$

```
T := RT[{x, y, z}, 10);
```

```
> T := RT[{x, y, z}, 10);
  SS := SSg[T, {x, y, z}];
  OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{8+2x+2y+4z}{1+2x+5y+6z}, \frac{10+5x+5y+z}{2+8x+10y+2z}, \frac{5+x+3y+9z}{2+2x+3y+5z}\right\}$$

$$SS := \{\{1.06290333, 0.9377247016, 1.55432057\}\}$$

$$OR := \{1.062903298, 0.9377247016, 1.554320564\} \quad (9)$$

```
> T := RT[{x, y, z}, 10);
  SS := SSg[T, {x, y, z}];
  OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{4+4x+5y+8z}{7+x+2y+4z}, \frac{2+2x+3y+2z}{6+4x+10y+4z}, \frac{4+7x+5y+z}{5+6x+3y+8z}\right\}$$

$$SS := \{\{-0.407802261, 0.1053037624, -0.588660169\}\}$$

$$OR := \{1.465633962, 0.4052527955, 0.7984346026\} \quad (10)$$

```
> T := RT[{x, y, z}, 10);
  SS := SSg[T, {x, y, z}];
  OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{9+9x+5y+3z}{1+9x+8y+10z}, \frac{7+3x+7y+3z}{8+4x+y+7z}, \frac{1+6x+8y+4z}{1+4x+10y+10z}\right\}$$

$$SS := \{\{0.97184506, 1.06957753, 0.7898538331\}\}$$

$$OR := \{0.9718450549, 1.069577460, 0.7898538331\} \quad (11)$$

```
> T := RT[{x, y, z}, 10);
  SS := SSg[T, {x, y, z}];
  OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{3+2x+6y+10z}{1+6x+7y+z}, \frac{2+9x+8y+10z}{1+10x+7y+7z}, \frac{5+10x+3y+5z}{5+3x+4y+9z}\right\}$$

$$SS := \{\{1.325780154, 1.140690727, 1.148503269\}\}$$

$$OR := \{1.325780154, 1.140690770, 1.148503241\} \quad (12)$$

```
> T := RT[{x, y, z}, 10);
  SS := SSg[T, {x, y, z}];
  OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{5+7x+5y+z}{6+10x+9y+z}, \frac{3+5x+10y+3z}{5+9x+5y+z}, \frac{3+2x+8y+6z}{2+8x+5y+9z}\right\}$$

$$SS := \{\{0.6701967325, 0.70731909, -1.360717531\}\}$$

$$OR := \{0.6817518377, 1.160411408, 0.8925748082\} \quad (13)$$

```
> T := RT[{x, y, z}, 10);
  SS := SSg[T, {x, y, z}];
  OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{1+2x+10y+10z}{9+6x+6y+10z}, \frac{9+4x+9y+8z}{8+7x+6y+8z}, \frac{9+8x+y+4z}{8+x+6y+3z}\right\}$$

$$SS := \{\{-1.198829777, -1.269393185, 0.9576002068\}\}$$

$$OR := \{0.7805913384, 1.064204489, 1.122236617\} \quad (14)$$

```
> T := RT[{x, y, z}, 10);
  SS := SSg[T, {x, y, z}];
  OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{10+9x+4y+2z}{8+2x+2y+7z}, \frac{9+6x+7y+7z}{2+2x+7y+8z}, \frac{3+x+y+4z}{10+9x+6y+9z}\right\}$$

$$SS := \{\{2.147953508, 1.765551012, 0.183964865\}\}$$

$$OR := \{2.147953508, 1.765551003, 0.1839648592\} \quad (15)$$

```
> T := RT[{x, y, z}, 10);
  SS := SSg[T, {x, y, z}];
  OR := ORB[T, {x, y, z}, {100., 200., 300.}, 1000, 1020][−1];
```

$$T := \left\{\frac{9+3x+8y+4z}{3+10x+10y+9z}, \frac{2+9x+4y+2z}{8+4x+3y+9z}, \frac{4+9x+5y+z}{2+9x+9y+4z}\right\}$$

$$T := \left[\frac{9+6x+6y+10z}{\ }, \frac{8+7x+6y+8z}{\ }, \frac{8+x+6y+3z}{\ }\right]$$

$$SS := [[-1.198829777,\ -1.269393185,\ 0.9576002068]]$$

$$OR := [0.7805913384,\ 1.064204489,\ 1.122236617] \quad \textbf{(14)}$$

```
> T := RT([x, y, z], 10);
  SS := SSg(T, [x, y, z]);
  OR := ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
```

$$T := \left[\frac{10+9x+4y+2z}{8+2x+2y+7z}, \frac{9+6x+7y+7z}{2+2x+7y+8z}, \frac{3+x+y+4z}{10+9x+6y+9z}\right]$$

$$SS := [[2.147953508,\ 1.765551012,\ 0.183964865]]$$

$$OR := [2.147953508,\ 1.765551003,\ 0.1839648592] \quad \textbf{(15)}$$

```
> T := RT([x, y, z], 10);
  SS := SSg(T, [x, y, z]);
  OR := ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
```

$$T := \left[\frac{9+3x+8y+4z}{3+10x+10y+9z}, \frac{2+9x+4y+2z}{8+4x+3y+9z}, \frac{4+9x+5y+z}{2+9x+9y+4z}\right]$$

$$SS := [[0.7361039687,\ -4.138289632,\ 0.3570779375]]$$

$$OR := [0.7980054820,\ 0.6521348398,\ 0.8311874429] \quad \textbf{(16)}$$

```
> T := RT([x, y, z], 10);
  SS := SSg(T, [x, y, z]);
  OR := ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
```

$$T := \left[\frac{2+6x+7y+4z}{4+4x+9y+2z}, \frac{9+5x+10y+7z}{9+2x+7y+z}, \frac{4+4x+6y+z}{8+8x+5y+6z}\right]$$

$$SS := [[0.6908427310,\ -1.278954485,\ -0.1802522306]]$$

$$OR := [0.9163797599,\ 1.506662542,\ 0.6486062268] \quad \textbf{(17)}$$

```
> T := RT([x, y, z], 10);
  SS := SSg(T, [x, y, z]);
  OR := ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
```

$$T := \left[\frac{3+3x+6y+5z}{7+9x+y+9z}, \frac{3+8x+5y+2z}{5+10x+y+9z}, \frac{10+8x+8y+2z}{4+3x+10y+4z}\right]$$

$$SS := [[0.5961395678,\ 0.5667117524,\ 1.312589912]]$$

$$OR := [0.5961395678,\ 0.5667117526,\ 1.312589911] \quad \textbf{(18)}$$

```
> T := RT([x, y, z], 10);
  SS := SSg(T, [x, y, z]);
  OR := ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
```

$$T := \left[\frac{6+9x+4y+2z}{10+2x+3y+3z}, \frac{4+4x+6y+7z}{5+2x+10y+5z}, \frac{2+10x+3y+6z}{5+9x+y+4z}\right]$$

$$SS := [[1.201403847,\ -1.51150507,\ 0.8188555238]]$$

$$OR := [1.243840795,\ 0.9893888310,\ 1.113186942] \quad \textbf{(19)}$$

```
> T := RT([x, y, z], 10);
  SS := SSg(T, [x, y, z]);
  OR := ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
```

$$T := \left[\frac{10+3x+4y+7z}{7+x+7y+z}, \frac{2+x+4y+7z}{9+8x+9y+6z}, \frac{7+8x+3y+3z}{2+2x+y+3z}\right]$$

$$SS := [[2.354038476,\ 0.495275931,\ 2.397315859]]$$

$$OR := [2.354038476,\ 0.4952759308,\ 2.397315918] \quad \textbf{(20)}$$

```
> T := RT([x, y, z], 10);
  SS := SSg(T, [x, y, z]);
  OR := ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
```

$$T := \left[\frac{5+4x+3y+10z}{1+8x+3y+8z}, \frac{8+7x+5y+6z}{7+x+5y+6z}, \frac{6+6x+4y+10z}{10+5x+9y+9z}\right]$$

$$SS := [[-0.1405103894,\ -1.439053522,\ 0.04523905468]]$$

$$OR := [1.062490794,\ 1.380637024,\ 0.7349321961] \quad \textbf{(21)}$$

```
>
```