

Ezra Chechik

Homework for Lecture 12 DMB.

1) For $a(n+1) = \frac{(1-a(n))(2-a(n))(3-a(n))}{12}$

→ Starting at $a(n) = 0.5$, we get $(1000 \rightarrow 1011$

$[0.2804250993, \text{repeated}]$, so, the ultimate period is 1 (1 sss).

→ 'high-low' with a 1st order non-linear recurrence with parameter, r .

$$a(n+1) = \frac{(1-a(n))(2-a(n))(3-a(n))}{r}$$

Estimates:

when $r > 6.90$, ultimate period 4

when $3.326 < r < 6.90$, ultimate period 2

when $2.572 < r < 3.326$, ultimate period 4

when $2.486 < r < 2.572$, ultimate period 8

2 and 3 on the next page :) ---->

2.

$$T := \left[\frac{28 + 29x + 14y}{18 + 27x + 15y}, \frac{8 + 10x + 14y}{12 + 9x + 13y} \right]$$

$$[[-0.8760268425, 0.08683426983]]$$

$$[1.178265185, 0.9462708737]$$

$$T := \left[\frac{28 + 10x + 17y}{14 + 12x + 26y}, \frac{23 + 15x + 7y}{5 + 26x + 19y} \right]$$

$$[[-1.885603367, 0.1078911478]]$$

$$[1.073232668, 0.9065557812]$$

$$T := \left[\frac{17 + 4x + 24y}{13 + 17x + 22y}, \frac{3 + 10x + 25y}{11 + 12x + 4y} \right]$$

$$[[-1.974903108, 0.4674720146]]$$

$$[0.9465730066, 2.126087388]$$

$$T := \left[\frac{6 + 27x + 11y}{28 + 9x + 20y}, \frac{2 + 10x + 16y}{13 + 18x + 3y} \right]$$

$$[[-1.838025242, 0.4725035034]]$$

$$[0.6501507388, 0.7716295616]$$

4/20 Fails

3.

$$T := \left[\frac{1 + 4x + 7y + 9z}{8 + 9x + 6y + 7z}, \frac{8 + 3x + 3y + 2z}{2 + x + 3y + 5z}, \frac{4 + 3x + 10y + z}{8 + 3x + 8y + 8z} \right]$$

$$[[-2.196782139, 0.9836751024, 0.565862225]]$$

$$[0.7341396111, 1.469087218, 0.7704402599]$$

$$T := \left[\frac{5+3x+8y+2z}{3+x+10y+7z}, \frac{6+6x+8y+10z}{4+5x+3y+8z}, \frac{1+10x+6y+8z}{10+9x+7y+8z} \right]$$

$$[[0.8129972410, -5.21105199, 1.359077411]]$$

$$[0.8753837616, 1.659352597, 0.7226374838]$$

$$T := \left[\frac{9+9x+6y+z}{4+8x+10y+8z}, \frac{5+6x+y+2z}{7+6x+10y+7z}, \frac{5+4x+6y+4z}{4+3x+7y+6z} \right]$$

$$[[-0.6442780499, 0.1156303359, -0.6331616691]]$$

$$[0.8760765688, 0.5241437821, 0.9641599271]$$

$$T := \left[\frac{4+3x+2y+8z}{8+3x+4y+9z}, \frac{2+2x+2y+3z}{2+9x+9y+z}, \frac{3+10x+2y+9z}{1+2x+8y+7z} \right]$$

$$[[0.1525151516, 0.239842100, -0.528265508]]$$

$$[0.7403459988, 0.5820505977, 1.425432461]$$

$$T := \left[\frac{1+x+9y+2z}{2+5x+3y+z}, \frac{4+x+6y+4z}{9+10x+4y+8z}, \frac{5+4x+6y+4z}{8+9x+9y+7z} \right]$$

$$[[0.4189176872, 0.5228048910, -2.372950169]]$$

$$[0.8325221443, 0.4059557999, 0.5638540590]$$

$$T := \left[\frac{10+6x+10y+z}{10+2x+9y+z}, \frac{9+4x+6y+2z}{8+6x+3y+5z}, \frac{8+10x+y+z}{7+7x+8y+5z} \right]$$

$$[[0.6128194771, -1.38455577, -1.519739760]]$$

$$[1.276338657, 0.9488953005, 0.8160706169]$$

$$T := \left[\frac{5+4x+3y+9z}{7+4x+5y+9z}, \frac{3+2x+2y+10z}{7+9x+6y+7z}, \frac{3+6x+2y+7z}{3+4x+8y+z} \right]$$

$$[[-3.523741494, -0.263305157, 0.9707992381]]$$

$$[0.8743039840, 0.7077603363, 1.459649531]$$

$$T := \left[\frac{10+3x+3y+10z}{1+4x+7y+2z}, \frac{7+5x+8y+10z}{6+7x+4y+2z}, \frac{9+9x+8y+2z}{5+10x+6y+6z} \right]$$

$$[[-1.622143010, -0.577505321, 0.9105637984]]$$

$$[1.500570157, 1.500645767, 1.038245610]$$

9/20 Fails

$$T := \left[\frac{7+5x+6y+7z}{1+5x+6y+6z}, \frac{6+4x+10y+10z}{5+9x+9y+5z}, \frac{10+9x+5y+5z}{2+4x+9y+10z} \right]$$

$$[[-1.189670387, -0.627019387, 0.9246734834]]$$

$$[1.342435875, 1.035927502, 1.166909602]$$