

# HW 13

$$1 \quad x_1 = \frac{x_1}{3+x_2} \quad x_2 = \frac{x_2}{2+x_1}$$

$$x_1 \left(1 - \frac{1}{3+x_2}\right) = 0 \quad x_2 \left(1 - \frac{1}{2+x_1}\right) = 0$$

$$x_1 = 0 \text{ or } x_2 = -2 \quad x_1 = -1 \text{ or } x_2 = 0$$

$$\therefore (0,0) \text{ or } (-1,-2)$$

$$1 = \frac{1}{3+x_2}$$

$$3+x_2 = 1$$

$$x_2 = -2$$

$$1 = \frac{1}{2+x_1}$$

$$2+x_1 = 1$$

$$x_1 = -1$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{3+x_2} & \frac{-x_1}{(3+x_2)^2} \\ \frac{-x_2}{(2+x_1)^2} & \frac{1}{(2+x_1)} \end{bmatrix}$$

$$f_1(\vec{x}) = x_1(3+x_2)^{-1}$$

$$f_2(\vec{x}) = x_2(2+x_1)^{-1}$$

$$J(0,0) = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$(1/3 - \lambda)(1/2 - \lambda) = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{3}$$

$(0,0)$  is stable

$$J(-1,-2) = \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$$

$$(1-\lambda)^2 - 2 = 0$$

$$1 - 2\lambda + \lambda^2 - 2 = 0$$

$$\lambda^2 - 2\lambda - 1 = 0$$

$$\lambda = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$(-1,-2)$  is not stable

2

$$x_1 = \frac{x_1}{3x_1 + 5x_2}$$

$$x_2 = \frac{x_2}{2x_1 + 7x_2}$$

$$x_1 \left(1 - \frac{1}{3x_1 + 5x_2}\right) = 0$$

$$x_2 \left(1 - \frac{1}{2x_1 + 7x_2}\right) = 0$$

$$x_1 = 0 \text{ or}$$

$$x_2 = 0 \text{ or}$$

$$3x_1 + 5x_2 = 1$$

$$2x_1 + 7x_2 = 1$$

$$6x_1 + 10x_2 = 2$$

$$2x_1 = \frac{4}{11}$$

$$-6x_1 + -21x_2 = -3$$

$$x_1 = \frac{2}{11}$$

$$-11x_2 = -1 \Rightarrow x_2 = \frac{1}{11}, x_1 = \frac{2}{11}$$

$$(0, \frac{1}{7}), (\frac{1}{3}, 0), (\frac{2}{11}, \frac{1}{11})$$

$$f_1(\vec{x}) = x_1 (3x_1 + 5x_2)^{-1} \quad f_2(\vec{x}) = x_2 (2x_1 + 7x_2)^{-1}$$

$$\frac{\partial f_1}{\partial x_1} = (3x_1 + 5x_2)^{-1} + x_1 \cdot -(3x_1 + 5x_2)^{-2} \cdot 3$$

$$= \frac{3x_1 + 5x_2 - 3x_1}{(3x_1 + 5x_2)^2}$$

$$= \frac{5x_2}{(3x_1 + 5x_2)^2}$$

$$\frac{\partial f_1}{\partial x_2} = x_1 \cdot -(3x_1 + 5x_2)^{-2} \cdot 5$$

$$= -5x_1$$

$$\frac{\partial f_2}{\partial x_2} = \frac{(3x_1 + 5x_2)^2}{(2x_1 + 7x_2)^{-1}} + x_2 \cdot -(2x_1 + 7x_2)^{-2} \cdot 7$$

$$2x_1$$

$$(2x_1 + 7x_2)^2$$

$$\frac{\partial f_2}{\partial x_1} = x_2 \cdot -(2x_1 + 7x_2)^{-2} \cdot 2$$

$$= -2x_2$$

$$(2x_1 + 7x_2)^2$$

$$J = \begin{bmatrix} \frac{5x_2}{(3x_1 + 5x_2)^2} & \frac{-5x_1}{(3x_1 + 5x_2)^2} \\ \frac{-2x_2}{(2x_1 + 7x_2)^2} & \frac{2x_1}{(2x_1 + 7x_2)^2} \end{bmatrix}$$

$$J(0, \frac{1}{5}) = \begin{bmatrix} 7/5 & 0 \\ -2/7 & 0 \end{bmatrix}$$

$$(\frac{7}{5} - \lambda)(-\lambda) = 0$$

$$\lambda = 0, \frac{7}{5} \Rightarrow \text{unstable}$$

$$J(\frac{1}{3}, 0) = \begin{bmatrix} 0 & -5/3 \\ 0 & 3/2 \end{bmatrix}$$

$$(-\lambda)(3/2 - \lambda) = 0$$

$$\lambda = \frac{3}{2}, 0 \Rightarrow \text{unstable}$$

$$J(\frac{2}{11}, \frac{1}{11}) = \begin{bmatrix} 5/11 & -1/11 \\ -2/11 & 4/11 \end{bmatrix}$$

$$\begin{aligned} (5/11 - \lambda)(4/11 - \lambda) - (\frac{1}{11})(\frac{2}{11}) &= 0 \\ (\frac{5}{11})(\frac{4}{11}) - (\frac{9}{11})\lambda + \lambda^2 - (\frac{1}{11})(\frac{2}{11}) &= 0 \end{aligned}$$

$$\begin{aligned} \lambda^2 - (\frac{9}{11})\lambda - \frac{18}{121} &= 0 \\ 11^2\lambda^2 - 99\lambda - 18 &= 0 \end{aligned}$$

$$\frac{99 \pm \sqrt{99^2 - 4(121)(-18)}}{2(121)}$$

$$= \frac{6}{11}, \frac{3}{11}$$

$\Rightarrow$  stable

