Homework for Lecture 12 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Oct. 20, 2025.

Subject: hw12

with an attachment hw8FirstLast.pdf and/or hw12FirstLast.txt

Using

`http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

1. What is the ultimate period of

`evalf(Orb((1-x)*(2-x)*(3-x)/12,x,0.5,1000,1010),10);`?

By Playing 'high-low', estimate the cut-offs r_1, r_2, r_3, r_4 such that

the recurrence

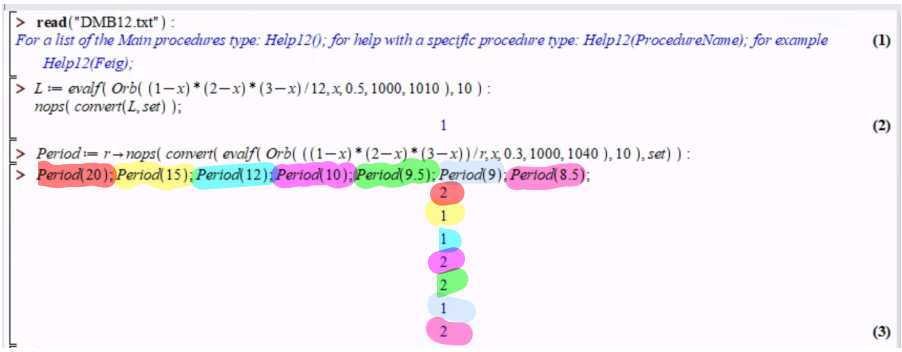
$$a(n+1) = (1-a(n))(2-a(n))(3-a(n))/r \quad ,$$

has

- ultimate period 1 for $r_1 < r$
- ultimate period 2 for $r_2 < r < r_1$
- ultimate period 4 for $r_3 < r < r_2$

(challenging!):

- ultimate period 8 for $r_4 < r < r_3$



2.

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

```
T:=RT([x,y],30); SSg(T,[x,y]); ORB(T,[x,y,z],[100., 200.],1000,1020);
```

Here the first command, `T:=RT([x,y],30);` generates a random rational transformation in R^2, the second command, SSg(T,[x,y]);, finds steady-states (i.e. fixed points of the transformation `T`, using

Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y],[100., 200.],1000,1020);` finds the long-term behavior.

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

`ORB(T,[x,y],[100., 200.],1000,1020)[-1];`

was not part of the output of

SSg(T,[x,y]);

3.

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

`T:=RT([x,y,z],10); SSg(T,[x,y,z]); ORB(T,[x,y,z],[100., 200.,300.],1000,1020);`

Here the first command, `T:=RT([x,y,z],10);` generates a random rational transformation in R^3, the second command, SSg(T,[x,y,z]);, finds steady-states (i.e. fixed points of the transformation `T`, using Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y,z],[100., 200.,300.],1000,1020);` finds the long-term behavior.

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

`ORB(T,[x,y,z],[100., 200.,300.],1000,1020)[-1];`

was not part of the output of

SSg(T,[x,y,z]);

2) SSg missed the final ORB point in trials: 4,5,9,14,16

Trial 4

```
> T := RT([x,y],30); SS := SSg(T,[x,y]); Traj := ORB(T, [x, y], [100., 200.], 1000, 1020) :
                    T := [ 24 + 20 x + 16 y    15 + 16 x + 26 y ]
                         [ ------------------, ---------------- ]
                         [ 17 + 4 x + 10 y     14 + 3 x + 2 y  ]
                   SS := [[-12.67551719, 4.600194074]]
> ORB(T, [x,y], [100., 200.], 1000, 1020)[-1];
                          [1.841642795, 6.604101582]
> Final := Traj[-1];
                   Final := [1.841642795, 6.604101582]
> Final6 := map(z -> eval(f[6](z), Final) : SS6 := [seq(map(z -> eval(f[6](z), SS[t]), t = 1 ..nops(SS))] : member(Final6, SS6);
                                        false
```

Trial 5

```
> T := RT([x,y],30); SS := SSg(T,[x,y]); Traj := ORB(T, [x, y], [100., 200.], 1000, 1020) :
                    T := [ 5 + 30 x + 6 y    29 + 16 x + 3 y ]
                         [ ---------------, ---------------- ]
                         [ 28 + 14 x + 14 y   8 + 16 x + 9 y ]
                   SS := [[0.3098791004, -2.572923186]]
> ORB(T, [x,y], [100., 200.], 1000, 1020)[-1];
                          [0.5331943342, 1.424327468]
> Final := Traj[-1];
                   Final := [0.5331943342, 1.424327468]
> Final6 := map(z -> eval(f[6](z), Final) : SS6 := [seq(map(z -> eval(f[6](z), SS[t]), t = 1 ..nops(SS))] : member(Final6, SS6);
                                        false
```

Trial 9

```
> T := RT([x,y],30); SS := SSg(T,[x,y]); Traj := ORB(T, [x, y], [100., 200.], 1000, 1020) :
                    T := [ 22 + 20 x + 14 y    13 + 23 x + 13 y ]
                         [ ----------------, ----------------- ]
                         [ 10 + 20 x + 4 y     5 + 28 x + y   ]
                   SS := [[0.03670629272, -1.612635093]]
> ORB(T, [x,y], [100., 200.], 1000, 1020)[-1];
                          [1.546388098, 1.326062000]
> Final := Traj[-1];
                   Final := [1.546388098, 1.326062000]
> Final6 := map(z -> eval(f[6](z), Final) : SS6 := [seq(map(z -> eval(f[6](z), SS[t]), t = 1 ..nops(SS))] : member(Final6, SS6);
                                        false
```

Trial 14

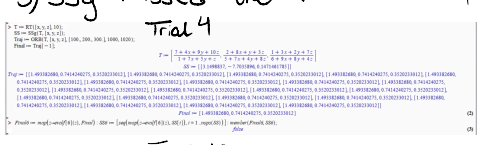
```
> T := RT([x,y],30); SS := SSg(T,[x,y]); Traj := ORB(T, [x, y], [100., 200.], 1000, 1020) :
                    T := [ 28 + 7 x + 17 y    20 + 18 x + 18 y ]
                         [ ----------------, ----------------- ]
                         [ 27 + 18 x + 14 y    19 + 12 x + 2 y ]
                   SS := [[-2.775900883, 0.9878386067]]
> ORB(T, [x,y], [100., 200.], 1000, 1020)[-1];
                          [0.9606053208, 2.202386800]
> Final := Traj[-1];
                   Final := [0.9606053208, 2.202386800]
> Final6 := map(z -> eval(f[6](z), Final) : SS6 := [seq(map(z -> eval(f[6](z), SS[t]), t = 1 ..nops(SS))] : member(Final6, SS6);
                                        false
```

Trial 16

```
> T := RT([x,y],30); SS := SSg(T,[x,y]); Traj := ORB(T, [x, y], [100., 200.], 1000, 1020) :
                    T := [ 2 + 8 x + 26 y    1 + 29 x + 3 y ]
                         [ ---------------, ---------------- ]
                         [ 2 + 5 x + 29 y    25 + 29 x + 2 y ]
                   SS := [[0.8917986486, -24.47974154]]
> ORB(T, [x,y], [100., 200.], 1000, 1020)[-1];
                          [1.057868270, 0.5882166446]
> Final := Traj[-1];
                   Final := [1.057868270, 0.5882166446]
> Final6 := map(z -> eval(f[6](z), Final) : SS6 := [seq(map(z -> eval(f[6](z), SS[t]), t = 1 ..nops(SS))] : member(Final6, SS6);
                                        false
```

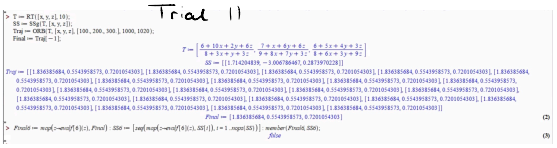
3) SSg missed the final ORB point in trials: 4,10,11,12,14,17,18,20

Trial 4

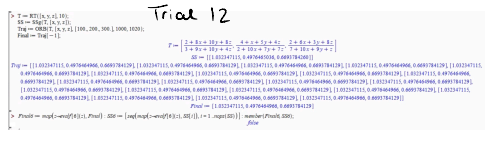


Trial 10

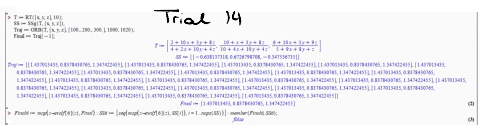
Trial 11



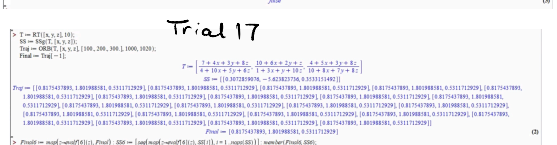
Trial 12



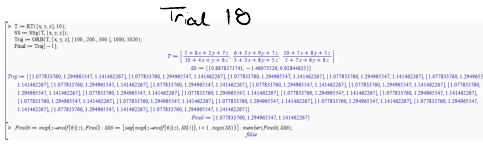
Trial 14



Trial 17



Trial 18



Trial 20