Homework for Lecture 12 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Oct. 20, 2025.

Subject: hw12

with an attachment hw8FirstLast.pdf and/or hw12FirstLast.txt

Using

`http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

1. What is the ultimate period of

`evalf(Orb((1-x)*(2-x)*(3-x)/12,x,0.5,1000,1010),10);`?

By Playing 'high-low', estimate the cut-offs r_1, r_2, r_3, r_4 such that

the recurrence

$$a(n+1) = (1-a(n))(2-a(n))(3-a(n))/r \quad ,$$

has

• ultimate period 1 for $r_1 < r$

• ultimate period 2 for $r_2 < r < r_1$

• ultimate period 4 for $r_3 < r < r_2$

(challenging!):

• ultimate period 8 for $r_4 < r < r_3$

2.

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

```
T:=RT([x,y],30); SSg(T,[x,y]); ORB(T,[x,y,z],[100., 200.],1000,1020);
```

Here the first command, `T:=RT([x,y],30);` generates a random rational transformation in R^2, the second command, SSg(T,[x,y]);, finds steady-states (i.e. fixed points of the transformation `T`, using

Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y],[100., 200.],1000,1020);` finds the long-term behavior.

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

`ORB(T,[x,y],[100., 200.],1000,1020)[-1];`

was not part of the output of

SSg(T,[x,y]);

3.

In the Maple package `http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt`

Run the following line 20 times

`T:=RT([x,y,z],10); SSg(T,[x,y,z]); ORB(T,[x,y,z],[100., 200.,300.],1000,1020);`

Here the first command, `T:=RT([x,y,z],10);` generates a random rational transformation in R^3, the second command, SSg(T,[x,y,z]);, finds steady-states (i.e. fixed points of the transformation `T`, using Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y,z],[100., 200.,300.],1000,1020);` finds the long-term behavior.

If all goes well, the ultimate beahvior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

`ORB(T,[x,y,z],[100., 200.,300.],1000,1020)[-1];`

was not part of the output of

SSg(T,[x,y,z]);

1)

```
> orb := evalf(Orb(((1-x)*(2-x)*(3-x))/12, x, 0.5, 1000, 1010), 10);
    nops(convert(orb, set));
orb := [0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993, 0.2804250993,
    0.2804250993, 0.2804250993, 0.2804250993]
                                    1
>
>
> orb := evalf(Orb(((1-x)*(2-x)*(3-x))/11, x, 0.5, 1000, 1010), 10);
    nops(convert(orb, set));
orb := [0.2953460398, 0.2953460398, 0.2953460398, 0.2953460398, 0.2953460398, 0.2953460398, 0.2953460398, 0.2953460398,
    0.2953460398, 0.2953460398, 0.2953460398]
                                    1
```

Ult period 1

From 12→11

```
> orb := evalf(Orb(((1-x)*(2-x)*(3-x))/8, x, 0.5, 1000, 1010), 10);
    nops(convert(orb, set));
orb := [0.3527798426, 0.3527798382, 0.3527798426, 0.3527798382, 0.3527798426, 0.3527798382, 0.3527798426, 0.3527798382,
    0.3527798426, 0.3527798382, 0.3527798426]
                                    2
> orb := evalf(Orb(((1-x)*(2-x)*(3-x))/4.2, x, 0.5, 1000, 1010), 10);
    nops(convert(orb, set));
orb := [1.634333755, -0.07542198862, 1.634333755, -0.07542198862, 1.634333755, -0.07542198862, 1.634333755,
    -0.07542198862, 1.634333755, -0.07542198862, 1.634333755]
                                    2
```

Ult period 2

From 8→4.2

```
> orb := evalf(Orb(((1-x)*(2-x)*(3-x))/4.3, x, 0.5, 1000, 1010), 10);
    nops(convert(orb, set));
orb := [1.602892462, -0.07778741962, 1.602892465, -0.07778741922, 1.602892462, -0.07778741962, 1.602892465,
    -0.07778741922, 1.602892462, -0.07778741962, 1.602892465]
                                    4
> orb := evalf(Orb(((1-x)*(2-x)*(3-x))/2.9, x, 0.5, 1000, 1010), 10);
    nops(convert(orb, set));
orb := [0.1315760575, 1.604916910, -0.1149704533, 2.532932385, 0.1315760575, 1.604916910, -0.1149704533, 2.532932385,
    0.1315760575, 1.604916910, -0.1149704533]
                                    4
```

Ult period 4

From 4.3→2.9

```
> orb := evalf(Orb(((1-x)*(2-x)*(3-x))/2.8, x, 0.5, 1000, 1010), 10);
    nops(convert(orb, set));
orb := [-0.1110071952, 2.605850937, 0.1369536087, 1.644099753, -0.1110071955, 2.605850943, 0.1369536085, 1.644099754,
    -0.1110071952, 2.605850937, 0.1369536087]
                                    8
> orb := evalf(Orb(((1-x)*(2-x)*(3-x))/2.5, x, 0.5, 1000, 1010), 10);
    nops(convert(orb, set));
orb := [0.1194622810, 1.907934982, -0.03651387116, 2.563880336, 0.1538353447, 1.778465010, -0.08426502028,
    2.788046846, 0.1194622810, 1.907934982, -0.03651387116]
                                    8
```

Ult period 8

From 2.8→2.5

2)

```
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1]; #a
                                [[1.788689382, 6.440736406]]
                                [1.788689382, 6.440736406]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[1.176137695, 0.9599975484]]
                                [1.176137695, 0.9599975484]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[1.747913138, 1.851973346]]
                                [1.747913138, 1.851973346]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[1.094536174, 1.069117841]]
                                [1.094536174, 1.069117840]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[-2.044168692, 0.1019738841]]
                                [0.6113881224, 0.9644715537]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[0.8917986486, -24.47974154]]
                                [1.057868270, 0.5882166446]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[1.293602558, 1.848660975]]
                                [1.293602558, 1.848660976]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                Page Break
                                [[-2.775900883, 0.9878386067]]
                                [0.9606053208, 2.202386800]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[0.03670629272, -1.612635093]]
                                [1.546388098, 1.326062000]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[0.3098791004, -2.572923186]]
                                [0.5331943342, 1.424327468]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[0.7778109416, -2.239220913]]
                                [0.8688141770, 0.8492487818]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[2.953221815, 0.9404217077]]
                                [2.953221815, 0.9404217077]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[-12.67551719, 4.600194074]]
                                [1.841642795, 6.604101582]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[1.486613682, 0.5587822436]]
                                [1.486613682, 0.5587822436]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[-1.885603367, 0.1078911478]]
                                [1.073232668, 0.9065557812]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[0.8862871228, 0.8735286408]]
                                [0.8862871228, 0.8735286405]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[1.472165575, 0.4583204366]]
                                [1.472165575, 0.4583204374]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[1.521847100, 0.1225005533]]
                                [1.521847100, 0.1225005300]
> T := RT([x, y], 30) : SSg(T, [x, y]); ORB(T, [x, y], [100., 200.1], 1000, 1020)[-1];
                                [[0.5779173450, 0.7478974050]]
```

Orb is not part of SSg

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3)

```
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[0.8062576542, -1.017114252, -1.362909194]]
                [0.9362259593, 1.622401984, 0.9102178310]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[0.6366202057, -0.8078823314, -0.1600703057]]
                [1.100252010, 1.620963429, 1.471856891]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                                    Page Break
                [[0.8738144044, 0.832498335, 2.331159891]]
                [0.8738144044, 0.8324983360, 2.331159891]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[1.37737067, 0.450244274, 0.7140851137]]
                [1.377370678, 0.4502442763, 0.7140851137]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[1.126417808, 2.296253471, 0.879121057]]
                [1.126417768, 2.296253471, 0.8791210894]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[-14.77823766, 0.1071776338, 2.375576351]]
                [2.526005871, 0.6390101404, 0.8784193884]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[-0.5212807504, -0.1840094872, 0.4100504131]]
                [1.143096277, 0.8211829882, 0.9837420900]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[0.3099499666, -2.57828685, -0.1663201]]
                [0.8494342113, 0.8646245541, 0.8572369656]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[1.593331893, 0.7835102102, 0.9764807147]]
                [1.593331889, 0.7835102102, 0.9764807155]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[1.095008924, -0.608706285, -1.315235870]]
                [1.217469326, 0.8180355851, 1.512641780]

> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[0.7150289971, -0.2498230485, -1.044141060]]
                [1.127893885, 0.3127135386, 1.377941716]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[-1.051635084, -0.5123073678, 0.1661001410]]
                [0.8224263947, 1.328440077, 1.097851157]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[2.19457810, 0.56639268, 0.7618555519]]
                [2.194578115, 0.5663926858, 0.7618555519]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[0.07733149158, -1.373850855, 0.611467259]]
                [1.571750821, 1.810656323, 0.7812502802]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[1.589113818, 2.464423150, 0.682645774]]
                [1.589113818, 2.464423177, 0.6826457609]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[0.2166542360, -6.065884561, 0.927963878]]
                [0.5838320410, 1.730001541, 0.7734327203]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[0.3923889269, 31.99639921, -22.60144481]]
                [1.708395383, 0.6526120078, 1.276780052]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[0.06560879449, 2.108625344, -6.409194012]]
                [0.1309386050, 0.9138043132, 1.675133269]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[1.462697005, 0.2517663327, -3.422728826]]
                [1.514248013, 0.6477917413, 0.9386779719]
> T := RT([x, y, z], 10) : SSg(T, [x, y, z]); ORB(T, [x, y, z], [100., 200., 300.], 1000, 1020)[-1];
                [[1.695350150, 1.739892283, 1.233679538]]
                [1.695350150, 1.739892288, 1.233679535]
```

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