

Homework for Lecture 12 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Oct. 20, 2025.

Subject: hw12

with an attachment hw8FirstLast.pdf and/or hw12FirstLast.txt

Using

<http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt>

1. What is the ultimate period of

`evalf(Orb((1-x)*(2-x)*(3-x)/12,x,0.5,1000,1010),10);?`

By Playing 'high-low', estimate the cut-offs r_1, r_2, r_3, r_4 such that

the recurrence

$$a(n+1) = (1 - a(n))(2 - a(n))(3 - a(n))/r \quad ,$$

has

- ultimate period 1 for $r_1 < r$
- ultimate period 2 for $r_2 < r < r_1$
- ultimate period 4 for $r_3 < r < r_2$

(challenging!):

- ultimate period 8 for $r_4 < r < r_3$

2.

In the Maple package <http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt>

Run the following line 20 times

```
T:=RT([x,y],30); SSg(T,[x,y]); ORB(T,[x,y,z],[100., 200.],1000,1020);
```

Here the first command, `T:=RT([x,y],30);` generates a random rational transformation in R^2 , the second command, `SSg(T,[x,y]);`, finds steady-states (i.e. fixed points of the transformation T , using

Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y],[100., 200.],1000,1020)`; finds the long-term behavior.

If all goes well, the ultimate behavior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

```
ORB(T,[x,y],[100., 200.],1000,1020)[-1];
```

was not part of the output of

```
SSg(T,[x,y]);
```

3.

In the Maple package <http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt>

Run the following line 20 times

```
T:=RT([x,y,z],10); SSg(T,[x,y,z]); ORB(T,[x,y,z],[100., 200.,300.],1000,1020);
```

Here the first command, `T:=RT([x,y,z],10)`; generates a random rational transformation in R^3 , the second command, `SSg(T,[x,y,z])`, finds steady-states (i.e. fixed points of the transformation `T`, using Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T,[x,y,z],[100., 200.,300.],1000,1020)`; finds the long-term behavior.

If all goes well, the ultimate behavior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

```
ORB(T,[x,y,z],[100., 200.,300.],1000,1020)[-1];
```

was not part of the output of

```
SSg(T,[x,y,z]);
```

Daniyal Chandhry HW 12

1. The ultimate period is $[0.2804250993, 0.2804250993 \dots]$

period 1 for $r_1 > 8.99$
 period 2 for $8.99 > r_2 > 4.37$
 period 4 for $4.37 > r_3 > 2.80$
 period 8 for $2.80 > r_4 > 2.49 \rightarrow$ I got lucky and guessed it on my first try!

} not sure why it's " $r >$ " rather than " $r <$ ", but I assume it's due to being $(\frac{1}{r})$, in the denominator

2. #5 $\left(\frac{29+11x+20y}{25+23x+8y}, \frac{9+17x+16y}{18+11x+20y} \right)$ SSg shared $0.553, -0.913$
 ORB shared $1.07, 1.0756$

#6 $\left(\frac{23+13x+8y}{17+6x+20y}, \frac{17+23x+6y}{25+21x+9y} \right)$ SSg: $0.055, -3.30$
 ORB: $1.106, 0.8$

#9 $\left(\frac{11+11x+30y}{6+8x+20y}, \frac{29+16x+11y}{23+9x+20y} \right)$ SSg: $-1.45, 0.222$
 ORB: $1.512, 1.09$

#14 $\left(\frac{14+30x+20y}{16+29x+19y}, \frac{3+9x+13y}{3+8x+7y} \right)$ SSg: $-0.13, 0.19$
 ORB: $1.01, 1.46$

#16 $\left(\frac{27+11x+25y}{20+30x+14y}, \frac{13+12x+9y}{18+21x+5y} \right)$ SSg: $-1.15, 0.05$
 ORB: $0.95, 0.75$

5 times of 20!

Pretty low compared to what I expected

Also: there was an erroneous "Z" in the code in the given problem, but no worries.

3. Since I expect more errors, here is a table for clarity
 coefficients from $L \rightarrow R, T \rightarrow B$ $\left(\frac{x+y+z}{x+y+z} \right)$

trial #	EQ 1	EQ 2	EQ 3	SSg	ORB
2	8 7 1 2 4 2 2 3	2 6 4 10 4 4 7 5	1 5 6 3 8 9 9 5	6.585, -5.09, 0.173	2.092, 1.066 0.497
5	1 6 10 9 1 3 5 10	3 5 9 5 1 5 2 8	6 2 8 5 9 1 2 10	-0.76, -0.18, 0.429	1.47, 1.773, 1.135
6	10 9 6 6 10 9 4 9	8 8 7 5 8 9 8 1	4 8 1 6 3 10 9 4	0.56, -3.75 7.93	1, 1.05, 0.68
8	4 3 10 10 9 2 9 4	2 8 4 3 9 4 9 5	1 2 9 9 4 2 6 7	0.1737, 0.3, -0.58	1.148, 0.068, 1.067
10	5 7 9 1 9 8 8 5	2 5 10 1 7 10 8 8	2 4 5 10 4 6 9 4	0.18, -0.6, -0.09	0.74, 0.38, 1.02
11	2 10 2 3 3 4 7 6	7 5 2 10 5 2 10 3	6 5 9 1 4 10 3 4	1.07, -0.68, -0.39	0.96, 1.14, 1.04
14	5 5 2 4 7 10 3 7	1 4 9 5 7 10 7 9	6 1 5 2 1 3 3 10	0.49, -1.36, 0.19	0.552, 0.455, 0.843
19	3 2 9 9 1 3 10 2	9 1 2 8 2 1 1 9	2 2 5 3 1 4 1 6	-9.14, 0.94, 5.94	1.35, 1.11, 0.99

8 out of 20 times

Wow