Homework for Lecture 11 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 13, 2025.

Subject: hw11



with an attachment hw11FirstLast.pdf

1. Read and understand, and be able to reproduce in an exam, the first part of

http://sites.math.rutgers.edu/~zeilberg/Bio25/L11.pdf

about period-doubling and advancement to chaos.

2. Find the potential steady-states of the third-order recurrence

$$a(n+3) = \frac{1+2a(n+2)+3a(n+1)+4a(n)}{2+3a(n+2)+4a(n+1)+5a(n)}$$

3. Convert the third-order recurrence

$$a(n+3) = \frac{1+2a(n+2)+3a(n+1)+4a(n)}{2+3a(n+2)+4a(n+1)+5a(n)}$$

to a first-order vector recurrence for an appropriate $\mathbf{x}(n)$ (you first have to define it).

4. By using SS in the Maple file

http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.pdf

find the two steady-states, in terms of the parameter r of the non-linear recurrence

$$a(n+1) = \frac{(1-2a(n))(1-3a(n))}{r}$$

By using SSS and experimenting with different r (playing high-low), estimate the cut-off r_0 such for $r < r_0$ there is no stable steady-state, but for $r > r_0$ one them is a stable steady-state.

- 4'. Optional challenge (5 dollars): Find the **exact** value of this cut-off r_0 .
- 5. (i) By playing 'high-low' estimate the number r_1 such that for $r < r_1$ the orbit tends to period 2 ultimate orbit until it starts having period 4.

- (ii) By playing 'high-low' estimate the number r_2 such that for $r < r_2$ the orbit tends to period 4 ultimate orbit until it starts having period 8.
- (iii) By playing 'high-low' estimate the number r_3 such that for $r < r_3$ the orbit tends to period 8 ultimate orbit until it starts having period 16.

2. Find the potential steady-states of the third-order recurrence

$$a(n+3) = \frac{1+2a(n+2)+3a(n+1)+4a(n)}{2+3a(n+2)+4a(n+1)+5a(n)}$$

$$Z = \frac{1+2z+3z+4z}{2+3z+4z+5z}$$

$$Z = \frac{1+0z}{2+12z}$$

$$2z+12z^2=1+0z$$

$$12z^2-7z-1=0$$

$$z = \frac{7\pm\sqrt{49-4(12)(1)}}{24(12)}$$

$$z = \frac{7\pm\sqrt{97}}{24}$$

$$z = \frac{1+0z}{2+12z}$$

$$10z = \frac{1+0z}{2+12z}$$

$$a(n+3) = \frac{1+2a(n+2)+3a(n+1)+4a(n)}{2+3a(n+2)+4a(n+1)+5a(n)} .$$

to a first-order vector recurrence for an appropriate $\mathbf{x}(n)$ (you first have to define it).

$$f(X_1 X_{2_1} X_3) = \frac{1 + 2X_1 + 3X_2 + 4X_3}{2 + 3X_1 + 4X_2 + 5X_3}$$

$$X(N+1) = F(X(n))$$

$$[X_1]X_2[X_3] \Rightarrow \frac{1+2\chi_1+3\chi_2+4\chi_3}{2+3\chi_1+4\chi_2+5\chi_3}$$

http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.pdf

find the two steady-states, in terms of the parameter r of the non-linear recurrence

$$a(n+1) = \frac{(1-2a(n))(1-3a(n))}{r}$$

$$f := \frac{(1-2\cdot x)\cdot (1-3\cdot x)}{r};$$

$$f := \frac{(1-2x)(1-3x)}{r}$$

$$\left[\frac{r}{12} + \frac{5}{12} + \frac{\sqrt{r^2 + 10\,r + 1}}{12}, \frac{r}{12} + \frac{5}{12} - \frac{\sqrt{r^2 + 10\,r + 1}}{12}\right]$$

By using SSS and experimenting with different r (playing high-low), estimate the cut-off r_0 such for $r < r_0$ there is no stable steady-state, but for $r > r_0$ one them is a stable steady-state.

$$f := \frac{(1-2\cdot x)\cdot (1-3\cdot x)}{3};$$

$$f := \frac{(1-2x)(1-3x)}{3}$$

$$\left[\frac{2}{3} - \frac{\sqrt{10}}{6}, \frac{2}{3} + \frac{\sqrt{10}}{6}\right]$$

$$f := \frac{(1-2\cdot x)\cdot (1-3\cdot x)}{4};$$

$$f := \frac{(1-2x)(1-3x)}{4}$$

$$\left[\frac{3}{4} - \frac{\sqrt{57}}{12}, \frac{3}{4} + \frac{\sqrt{57}}{12}\right]$$

$$\left[\frac{3}{4} - \frac{\sqrt{57}}{12}\right]$$

Yo ~ 3.4

3 4 Y 0 24

$$f := \frac{(1-2\cdot x)\cdot (1-3\cdot x)}{3.4};$$

$$f := 0.2941176471 (1 - 2x) (1 - 3x)$$

[0.1313759297, 1.268624070]

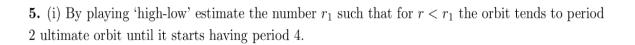
[]

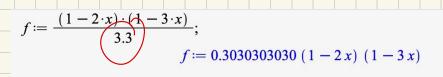
$$f := \frac{(1-2\cdot x)\cdot (1-3\cdot x)}{3.5};$$

$$f := 0.2857142857 (1 - 2x) (1 - 3x)$$

[0.1294815004, 1.287185166]

[0.1294815004]





-> period

Orb(*f*, *x*, .5, 1000, 1010)

-).2377713552, 0.04556197821, 0.2377713552, 0.04556197821, 0.2377713552, 0.04556197821, 0.2377713552, 0.04556197821, 0.2377713552, 0.04556197821, 0.2377713552]

$$f := \frac{(1 - 2 \cdot x) \cdot (1 - 3 \cdot x)}{3.4};$$

$$f := 0.2941176471 (1 - 2x) (1 - 3x)$$

Orb (*f*, *x*, .5, 1000, 1010)

0.1804738036, 0.08619286610, 0.1804738031, 0.08619286651, 0.1804738026, 0.08619286692, 0.1804738021, 0.08619286737, 0.1804738016, 0.08619286778, 0.1804738011]

apeno dic?