Homework For Lecture 11

1) when $r > 3$ in $a(n+1) = r\,a(n)(1-a(n))$

you get alternating → chaos.

→ when $r = 3.2$, result of $x \to rx(1-x)$ twice seems to have stable steady states.

$f_1(x) = f(f(x)) = -r^3x^4 + 2r^3x^3 - r^3x^2 - r^2x^2 + r^2x$

→ But, for $3.449487 < r < 3.5440903$, we get $2^2 = 4$ period orbit. The rate of period dubling is δ, Feigenbaum Const.

$\delta = 4.6692\ldots$ Before you know it as $r > 3$, $r \uparrow$, you get chaos.

2) potential steady states:

$$a(n+3) = \frac{1 + 2a(n+2) + 3a(n+1) + 4a(n)}{2 + 3a(n+2) + 4a(n+1) + 5a(n)}$$

$$Z = \frac{1 + 2Z + 3Z + 4Z}{2 + 3Z + 4Z + 5Z} \to Z = \frac{9Z + 1}{12Z + 2}$$

$$12Z^2 + 2Z - 9Z - 1 = 0 \to 12Z^2 - 7Z - 1 = 0$$

using maple solve →

$$Z_1 = 7/24 + \sqrt{97}/24,\ Z_2 = 7/24 - \sqrt{97}/24$$

3) $x(n+1) = F(x(n))$

$$x(n) = \begin{bmatrix} a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix}$$

$$F(x_1, x_2, x_3) = \begin{bmatrix} \dfrac{1 + 2x_1 + 3x_2 + 4x_3}{2 + 3x_1 + 4x_2 + 5x_3} \\ x_1 \\ x_2 \end{bmatrix}$$

4) $$\frac{(1-2x)(1-3x)}{r} = \frac{6x^2 - 5x + 1}{r}$$

$F :=$

→ $SS(f, x)$, we get

$$\left[\frac{r}{12} + \frac{5}{12} + \frac{\sqrt{r^2 + 10r + 1}}{12}, \frac{r}{12} + \frac{5}{12} - \frac{\sqrt{r^2 + 10r - 1}}{12} \right]$$

Finding Cutoff using trial / error.

Cutoff $= -9.9$

if $r_0 < -9.9$, 1 SSS

if $r_0 > -9.9$, no SSS

5) $ff := f(f(x))$

i) around 3.42 , using

ii) 3.784

iii) 3.81