**Homework for Lecture 11 of Dr. Z.'s Dynamical Models in Biology class**

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 13, 2025.

Subject: hw11

with an attachment hw11FirstLast.pdf

**1.** Read and understand, and be able to reproduce in an exam, the first part of

`http://sites.math.rutgers.edu/~zeilberg/Bio25/L11.pdf`

about period-doubling and advancement to chaos.

**2.** Find the potential steady-states of the third-order recurrence

$$a(n+3) = \frac{1+2a(n+2)+3a(n+1)+4a(n)}{2+3a(n+2)+4a(n+1)+5a(n)}$$

$z = \frac{1+2z+3z+4z}{2+3z+4z+5z} \Rightarrow \frac{1+9z}{2+12z} \rightarrow z(2+12z)=1+9z$

$12z^2-7z-1=0$

$z = \frac{7 \pm \sqrt{97}}{24}$

**3.** Convert the third-order recurrence

$$a(n+3) = \frac{1+2a(n+2)+3a(n+1)+4a(n)}{2+3a(n+2)+4a(n+1)+5a(n)}$$

$$\cdot \begin{bmatrix} \frac{1+2x_1+3x_2+4x_3}{2+3x_1+4x_2+5x_3} \\ x_1 \\ x_2 \end{bmatrix}$$

to a first-order vector recurrence for an appropriate $\mathbf{x}(n)$ (you first have to define it).

**4.** By using `SS` in the Maple file * See next page *

`http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.pdf`

find the two steady-states, in terms of the parameter $r$ of the non-linear recurrence

$$a(n+1) = \frac{(1-2\,a(n))(1-3\,a(n))}{r} \quad .$$

By using `SSS` and experimenting with different $r$ (playing *high-low*), estimate the cut-off $r_0$ such for $r < r_0$ there is no stable steady-state, but for $r > r_0$ one them is a stable steady-state.

**4'.** Optional challenge (5 dollars): Find the **exact** value of this cut-off $r_0$.

**5.** (i) By playing 'high-low' estimate the number $r_1$ such that for $r < r_1$ the orbit tends to period 2 ultimate orbit until it starts having period 4.

* See next page *

(ii) By playing 'high-low' estimate the number $r_2$ such that for $r < r_2$ the orbit tends to period 4 ultimate orbit until it starts having period 8.

(iii) By playing 'high-low' estimate the number $r_3$ such that for $r < r_3$ the orbit tends to period 8 ultimate orbit until it starts having period 16.

4)

```
> SS( (1-2*x)*(1-3*x)/r,x );
```

$$\left[\frac{r}{12}+\frac{5}{12}+\frac{\sqrt{r^2+10r+1}}{12},\ \frac{r}{12}+\frac{5}{12}-\frac{\sqrt{r^2+10r+1}}{12}\right] \quad (2)$$

```
> r := 7.0 :
> SSS( (1-2*x)*(1-3*x)/r,x );
```

[0.08712907084] (3)

```
> r := 7.5 :
> SSS( (1-2*x)*(1-3*x)/r,x );
```

[0.08333333332] (4)

```
> r := 8.0 :
> SSS( (1-2*x)*(1-3*x)/r,x );
```

[0.07986711843] (5)

5)

```
> f := (1-2*x)*(1-3*x)/r :
> x0 := 0.10 :
> K1 := 1200 :
> K2 := 1215 :
> r := 7.7 :
  evalf( Orb( f,x,x0,K1,K2 ) );
```

[0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671] (2)

```
> r := 7.9 :
  evalf(Orb(f,x,x0,K1,K2));
```

[0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671] (3)

```
> f := (1-2*x)*(1-3*x)/r :
> x0 := 0.10 :
> K1 := 1200 :
> K2 := 1215 :
> r := 8.2 :
  evalf( Orb( f,x,x0,K1,K2 ) );
```

[0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671] (2)

```
> r := 8.3 :
  evalf( Orb( f,x,x0,K1,K2 ) );
```

[0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671] (3)

```
> f := (1-2*x)*(1-3*x)/r :
> x0 := 0.10 :
> K1 := 1200 :
> K2 := 1215 :
> r := 8.45 :
  evalf( Orb( f,x,x0,K1,K2 ) );
```

[0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671] (2)

```
> r := 8.55 :
  evalf( Orb( f,x,x0,K1,K2 ) );
```

[0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671, 0.08121696674, 0.08121696671] (3)