② $c = \dfrac{1+2c+3c+4c}{2+3c+4c+5c} = \dfrac{1+9c}{2+12c}$

$2c + 12c^2 = 1+9c$ $\qquad 12c^2 - 7c - 1 = 0$

$\dfrac{7 \pm \sqrt{97}}{24}$ $\qquad c_1 = 0.7 \qquad c_2 = -0.118$

② $\begin{bmatrix} x_1(n) \\ x_2(n) \\ x_3(n) \end{bmatrix} = \begin{bmatrix} a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix}$ $\quad a(n+3) = \ldots$

$x(n+1) = f(x(n)) = \begin{bmatrix} \dfrac{1+2a(n+2)+3a(n+1)+4an}{2+3x_1(n)+4x_2(n)+5x_3(n)} \\ x_1(n) \\ x_2(n) \end{bmatrix}$

④ $r<6$ with $a=1/3$ stable

$r=6$ bifurcation

$r>6$ both steady states exist $a^* = 1/3$ unstable
$a = \frac{r-6}{2r}$ = stable

4) $(5+r)^2 - 24 = 0$
$r^2 + 10r + 1 = 0$
$f(1/3) = 1$ $\quad \begin{cases} r^2 - r + 36 = 0 \\ r^2 + r - 36 = 0 \end{cases}$ $\quad \dfrac{-1 + \sqrt{145}}{2} = 5.5 \approx 6$

$r_0 = 6$

5. i. $r = 9.15$
$r < 9.15 = 2$
$r > 9.15 = 4$

ii. $r = 9.9$
$r < 9.9 = 4$
$r > 9.9 = 8$

iii. $r = 10.35$
$r < 10.35 = 8$
$r > 10.35 = 16$