Homework for Lecture 11 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Oct. 13, 2025.

Subject: hw11

with an attachment hw11FirstLast.pdf

1. Read and understand, and be able to reproduce in an exam, the first part of

`http://sites.math.rutgers.edu/~zeilberg/Bio25/L11.pdf`

about period-doubling and advancement to chaos.

2. Find the potential steady-states of the third-order recurrence

$$a(n+3) = \frac{1 + 2a(n+2) + 3a(n+1) + 4a(n)}{2 + 3a(n+2) + 4a(n+1) + 5a(n)} \quad .$$

3. Convert the third-order recurrence

$$a(n+3) = \frac{1 + 2a(n+2) + 3a(n+1) + 4a(n)}{2 + 3a(n+2) + 4a(n+1) + 5a(n)} \quad .$$

to a first-order vector recurrence for an appropriate $\mathbf{x}(n)$ (you first have to define it).

4. By using `SS` in the Maple file

`http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.pdf`

find the two steady-states, in terms of the parameter r of the non-linear recurrence

$$a(n+1) = \frac{(1 - 2\,a(n))(1 - 3\,a(n))}{r} \quad .$$

By using `SSS` and experimenting with different r (playing *high-low*), estimate the cut-off r_0 such for $r < r_0$ there is no stable steady-state, but for $r > r_0$ one them is a stable steady-state.

4'. Optional challenge (5 dollars): Find the **exact** value of this cut-off r_0.

5. (i) By playing 'high-low' estimate the number r_1 such that for $r < r_1$ the orbit tends to period 2 ultimate orbit until it starts having period 4.

(ii) By playing 'high-low' estimate the number r_2 such that for $r < r_2$ the orbit tends to period 4 ultimate orbit until it starts having period 8.

(iii) By playing 'high-low' estimate the number r_3 such that for $r < r_3$ the orbit tends to period 8 ultimate orbit until it starts having period 16.

Daniyal Chaudhry HW11

① Reviewed ✓

② $a(n+3) = \dfrac{1+2a(n+2)+3a(n+1)+4a(n)}{2+3a(n+2)+4a(n+1)+5a(n)} = \dfrac{1+2z+3z+4z}{2+3z+4z+5z} = \dfrac{1+9z}{2+12z} \Rightarrow z = \dfrac{1+9z}{2+12z}$

$\Downarrow$

$2z + 12z^2 - 9z - 1 = 0$

$12z^2 - 7z - 1 = 0$

potential steady states $\boxed{z = \dfrac{7 \pm \sqrt{49+48}}{24}}$ $\Longleftarrow$

③ $\vec{x}(n) = \begin{bmatrix} a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix} \rightarrow F\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} \frac{1+2x_1+x_2+4x_3}{2+3x_1+4x_2+5x_3} \\ x_1 \\ x_2 \end{bmatrix}$

④ in terms of r, we have the two SS: $\dfrac{r}{12} + \dfrac{5}{12} \pm \dfrac{\sqrt{r^2+10r+1}}{12}$

r_0 for 1 SSS: ≈ 3.4305

⑤ i) ~2.09 , the ultimate orbit shifts from period 2 to 4
ii) ~ 1.92 , shifts from 4 → 8
iii) ~ 1.38 , from 8 → 16