

## Homework for Lecture 10 of Dr. Z.'s Dynamical Models in Biology class

Email the answers ( as .pdf file ) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Oct. 13, 2025.

Subject: hw8

with an attachment hw8FirstLast.pdf and/or hw8FirstLast.txt

Using

<http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB10.txt>

**1.** Using procedure `ExactPD` find the probability of winning, and expected duration of the game in a Gambler's Ruin with max. capital 40 dollars, starting capital 20 dollars and prob. of winning a dollar at any one round  $\frac{19}{40}$ .

Now using `EstPD`, estimate it with (i)  $K=100$ , (ii)  $K=1000$  (iii)  $K=5000$  (Warning: this may take some time) Which of them gives the best estimate? Can you explain why?

**2.**

Plot `ForPD(n,100,p)[1]` from  $p=0.45$  to  $p=0.5$  for  $n=10,20,30,40,50,60,70,80,90$ .

1. Using procedure **ExactPD** find the probability of winning, and expected duration of the game in a Gambler's Ruin with max. capital 40 dollars, starting capital 20 dollars and prob. of winning a dollar at any one round  $\frac{19}{40}$ .

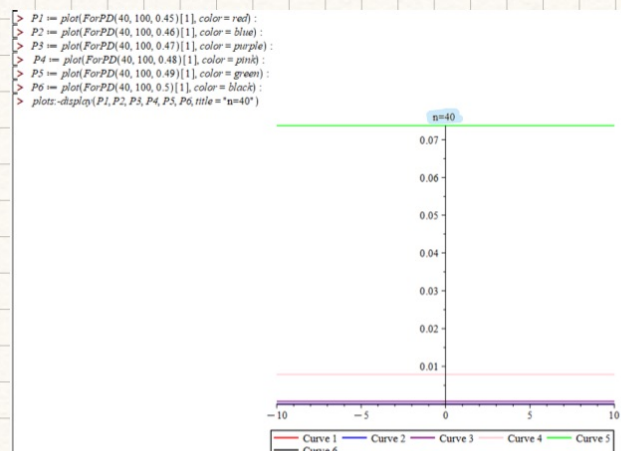
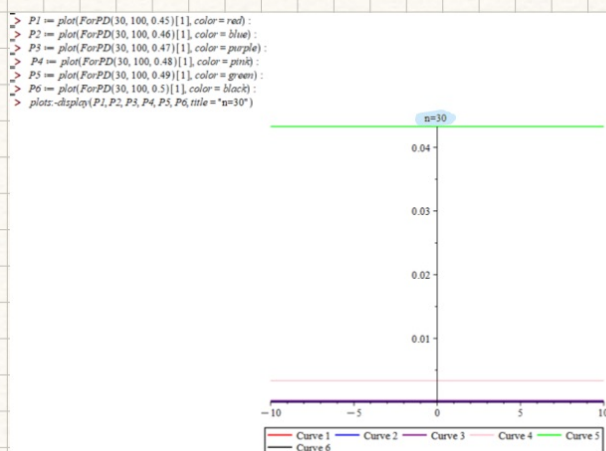
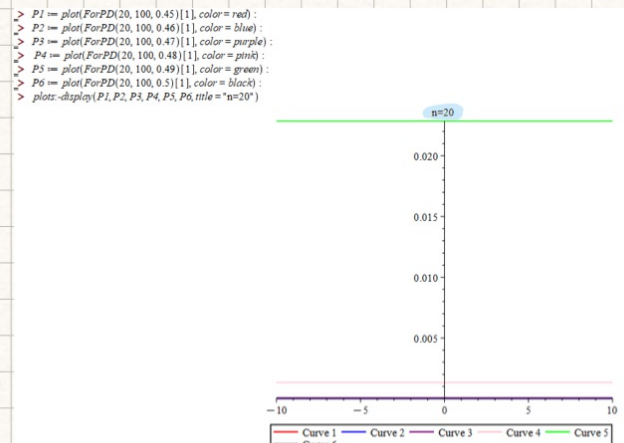
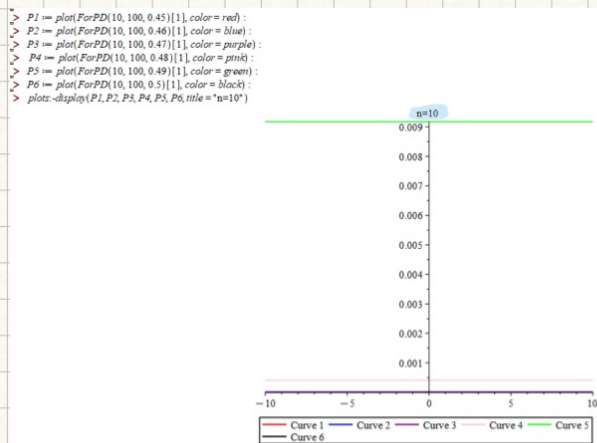
Now using **EstPD**, estimate it with (i)  $K=100$ , (ii)  $K=1000$  (iii)  $K=5000$  (Warning: this may take some time) Which of them gives the best estimate? Can you explain why?

```
> ExactPD(20, 40, 19/40)
= [0.1190277811, 304.7777751]
> EstPD(20, 40, 19/40, 100)
= [0.1100000000, 292.0600000]
> EstPD(20, 40, 19/40, 1000)
= [0.1310000000, 308.3000000]
> EstPD(20, 40, 19/40, 5000)
= [0.1234000000, 307.1728000]
```

| PROB OF WINNING | DURATION |
|-----------------|----------|
| 0.119           | 304 rds  |
| 0.110           | 292 rds  |
| 0.131           | 308 rds  |
| 0.123           | 307 rds  |

$K=5000$  has the best estimate b/c  $0.123 \approx 0.119$  and 307 is closest to 304

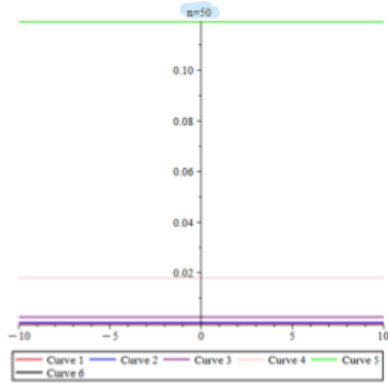
## Problem #2



```

> P1 := plot(ForPD(50, 100, 0.45)[1], color = red) :
> P2 := plot(ForPD(50, 100, 0.46)[1], color = blue) :
> P3 := plot(ForPD(50, 100, 0.47)[1], color = purple) :
> P4 := plot(ForPD(50, 100, 0.48)[1], color = pink) :
> P5 := plot(ForPD(50, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(50, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=50")

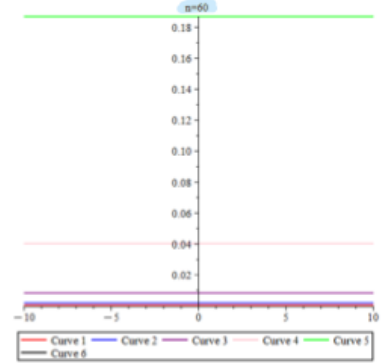
```



```

> P1 := plot(ForPD(60, 100, 0.45)[1], color = red) :
> P2 := plot(ForPD(60, 100, 0.46)[1], color = blue) :
> P3 := plot(ForPD(60, 100, 0.47)[1], color = purple) :
> P4 := plot(ForPD(60, 100, 0.48)[1], color = pink) :
> P5 := plot(ForPD(60, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(60, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=60")

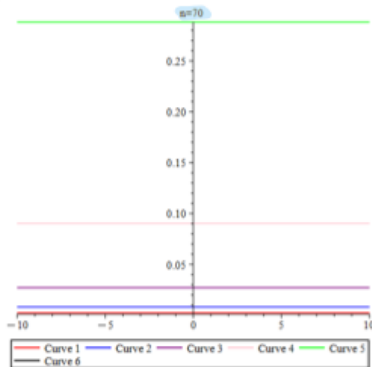
```



```

> P1 := plot(ForPD(70, 100, 0.45)[1], color = red) :
> P2 := plot(ForPD(70, 100, 0.46)[1], color = blue) :
> P3 := plot(ForPD(70, 100, 0.47)[1], color = purple) :
> P4 := plot(ForPD(70, 100, 0.48)[1], color = pink) :
> P5 := plot(ForPD(70, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(70, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=70")

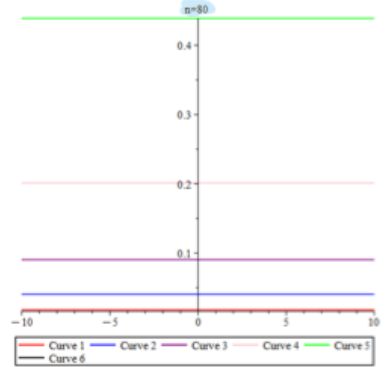
```



```

> P1 := plot(ForPD(80, 100, 0.45)[1], color = red) :
> P2 := plot(ForPD(80, 100, 0.46)[1], color = blue) :
> P3 := plot(ForPD(80, 100, 0.47)[1], color = purple) :
> P4 := plot(ForPD(80, 100, 0.48)[1], color = pink) :
> P5 := plot(ForPD(80, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(80, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=80")

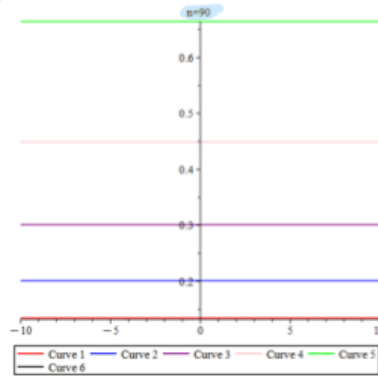
```



```

> P1 := plot(ForPD(90, 100, 0.45)[1], color = red) :
> P2 := plot(ForPD(90, 100, 0.46)[1], color = blue) :
> P3 := plot(ForPD(90, 100, 0.47)[1], color = purple) :
> P4 := plot(ForPD(90, 100, 0.48)[1], color = pink) :
> P5 := plot(ForPD(90, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(90, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=90")

```



1. Using procedure **ExactPD** find the probability of winning, and expected duration of the game in a Gambler's Ruin with max. capital 40 dollars, starting capital 20 dollars and prob. of winning a dollar at any one round  $\frac{19}{40}$ .

Now using **EstPD**, estimate it with (i) K=100, (ii) K=1000 (iii) K=5000 (Warning: this may take some time) Which of them gives the best estimate? Can you explain why?

```
> ExactPD(20, 40, 19/40)
[0.1190277811, 304.7777751]
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> EstPD(20, 40, 19/40, 1000)
[0.1310000000, 308.3000000]
=
> EstPD(20, 40, 19/40, 5000)
[0.1234000000, 307.1728000]
=
```

prob. of winning: 0.119 Duration: 304 rds

prob of winning: 0.110 Duration: 292 rds

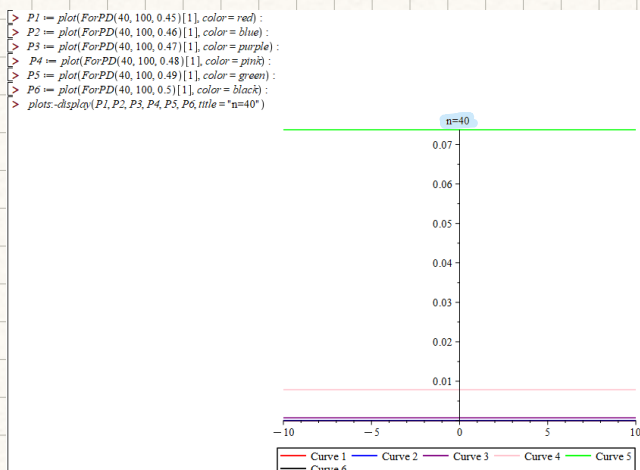
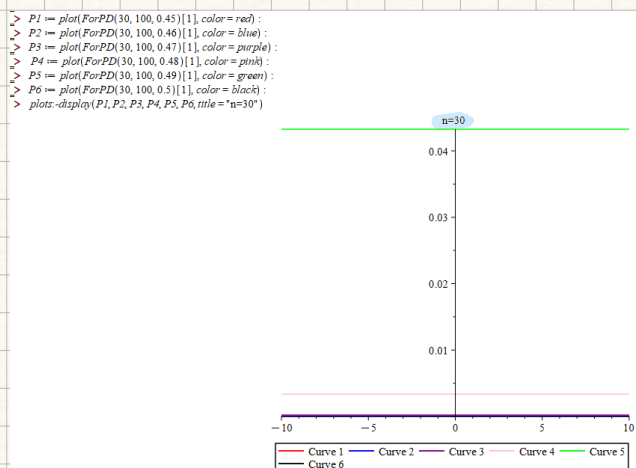
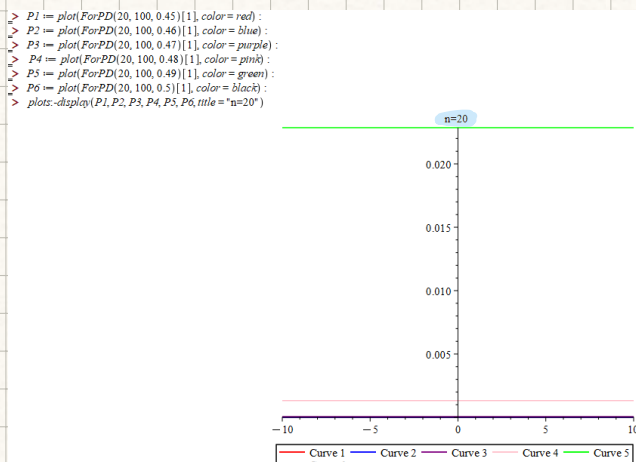
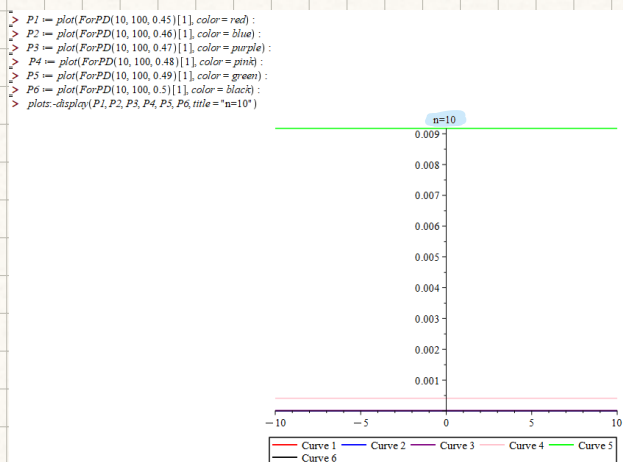
prob of winning: 0.131 Duration: 308 rds

prob of winning: 0.123 Duration: 307 rds

K=5000 has the best estimate bc 0.123 % 0.119 and 307 is closest to 304.

2.

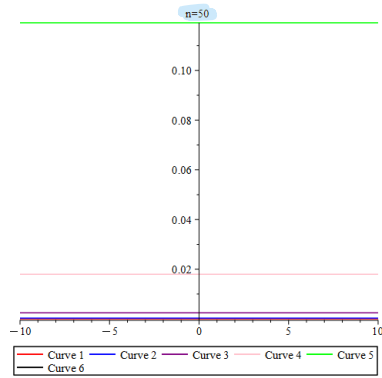
Plot ForPD(n, 100, p) [1] from p=0.45 to p=0.5 for n=10, 20, 30, 40, 50, 60, 70, 80, 90.



```

> P1 := plot(ForPD(50, 100, 0.45)[1], color = red) :
> P2 := plot(ForPD(50, 100, 0.46)[1], color = blue) :
> P3 := plot(ForPD(50, 100, 0.47)[1], color = purple) :
> P4 := plot(ForPD(50, 100, 0.48)[1], color = pink) :
> P5 := plot(ForPD(50, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(50, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=50")

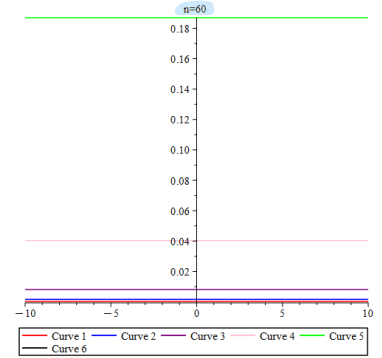
```



```

> P1 := plot(ForPD(60, 100, 0.45)[1], color = red) :
> P2 := plot(ForPD(60, 100, 0.46)[1], color = blue) :
> P3 := plot(ForPD(60, 100, 0.47)[1], color = purple) :
> P4 := plot(ForPD(60, 100, 0.48)[1], color = pink) :
> P5 := plot(ForPD(60, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(60, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=60")

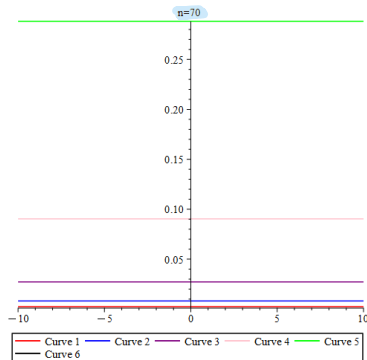
```



```

> P1 := plot(ForPD(70, 100, 0.45)[1], color = red) :
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> P5 := plot(ForPD(70, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(70, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=70")

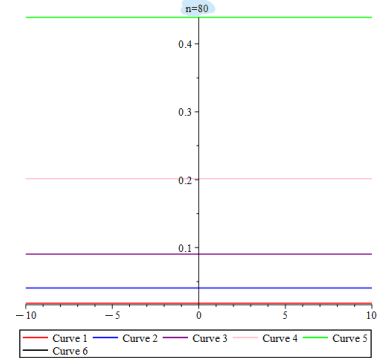
```



```

> P1 := plot(ForPD(80, 100, 0.45)[1], color = red) :
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> P4 := plot(ForPD(80, 100, 0.48)[1], color = pink) :
> P5 := plot(ForPD(80, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(80, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=80")

```



```

> P1 := plot(ForPD(90, 100, 0.45)[1], color = red) :
> P2 := plot(ForPD(90, 100, 0.46)[1], color = blue) :
> P3 := plot(ForPD(90, 100, 0.47)[1], color = purple) :
> P4 := plot(ForPD(90, 100, 0.48)[1], color = pink) :
> P5 := plot(ForPD(90, 100, 0.49)[1], color = green) :
> P6 := plot(ForPD(90, 100, 0.5)[1], color = black) :
> plots:-display(P1, P2, P3, P4, P5, P6, title = "n=90")

```

