

1. nth terms of the sequence

$$X_n = X_{n-1} + 2X_{n-2} - X_{n-3} \quad n \geq 3$$

$$X_0 = 1 \quad X_1 = 2 \quad X_2 = -1.$$

$$X_3 = X_2 + 2X_1 - X_0 = -1 + 2(2) - 1 = -1 + 4 = 2.$$

$$X_4 = X_3 + X_2 + -X_1 = 2 + (-1) - 2 = -1.$$

$$X_5 = X_4 + X_3 + -X_2 = -1 + 2 - (-1) = 2.$$

$$X_6 = X_5 + X_4 - X_3 = 2 + (-1) - 2 = -1.$$

2. Solve explicitly the recurrence equation

$$X_n = 5X_{n-1} - 6X_{n-2} \text{ with initial conditions } X_0 = 0, X_1 = 1.$$

$$z^2 = 5z - 6.$$

$$z^2 - 5z + 6 = 0.$$

$$0 = (z-2)(z-3).$$

$$z_1 = 2 \text{ or } z_2 = 3.$$

$$X_n = A \cdot 2^n + B \cdot 3^n$$

When  $n=0$ ,

$$X_0 = A \cdot 2^0 + B \cdot 3^0 = A + B = 0 \quad A = -B.$$

When  $n=1$ ,

$$X_1 = A \cdot 2^1 + B \cdot 3^1 = 2A + 3B = 1. \quad 2(-B) + 3B = 1. \quad B = 1.$$

$$X_n = 3^n - 2^n$$

$$A = -1.$$

Let  $B_n$  be the expected number of females born at time  $n$ .

$$B_n = p_1 B_{n-1} + p_2 B_{n-2}$$

when  $n=0$   $B_0 = c_0$

when  $n=1$   $B_1 = c_1$

$n=2$

~~$$B_2 = p_1 B_1 + p_2 B_0$$~~

$$B_2 = p_1 B_1 + p_2 B_0 = p_1 c_1 + p_2 c_0$$

$$B_3 = p_1 B_2 + p_2 B_1 = p_1 (p_1 c_1 + p_2 c_0) + p_2 c_1$$

$$= p_1^2 c_1 + p_1 p_2 c_0 + p_2 c_1$$

$$B_4 = p_1 B_3 + p_2 B_2 = (p_1^3 c_1 + p_1^2 p_2 c_0 + p_1 p_2 c_1)$$

$$+ p_2 (p_1 c_1 + p_2 c_0)$$