

HW 1

① $X_n = X_{n-1} + 2X_{n-2} - X_{n-3}, \quad n \geq 3, \quad X_0 = 1, X_1 = 2, X_2 = -1$

$$X_3 = X_2 + 2X_1 - X_0 = -1 + 4 - 1 = 2$$

$$X_4 = X_3 + 2X_2 - X_1 = 2 + (-2) - 2 = -2$$

$$X_5 = X_4 + 2X_3 - X_2 = -2 + 4 + 1 = 3$$

$$X_6 = 3 + (-4) - 2 = -3$$

$$X_7 = -3 + 6 + 2 = 5$$

$$X_8 = 5 - 6 - 3 = -4$$

② $X_n = 5X_{n-1} - 6X_{n-2}, \quad X_0 = 0, X_1 = 1$

$$z^2 = 5z - 6 \rightarrow z^2 - 5z + 6 \rightarrow (z-3)(z-2) = 0 \quad z = 3, 2$$

Gen Soln: $C_1 3^n + C_2 2^n$

$$\begin{cases} 0 = C_1 + C_2 \\ 1 = 3C_1 + 2C_2 \end{cases}$$

$$1 = -3C_2 + 2C_2 \rightarrow -1 = C_2, \quad 1 = C_1$$

$$\boxed{X_n = 3^n - 2^n}$$

③ $X_n = P_1 X_{n-1} + P_2 X_{n-2} + P_3 X_{n-3}, \quad n \geq 3$

$$C_3 = P_1 C_2 + P_2 C_1 + P_3 C_0$$

$$C_4 = P_1 C_3 + P_2 C_2 + P_3 C_1$$

$$= P_1(P_1 C_2 + P_2 C_1 + P_3 C_0) + P_2 C_2 + P_3 C_1$$

$$= (P_1^2 + P_3) C_2 + (P_1 P_2 + P_3) C_1 + (P_1 P_3) C_0$$