1. Compute the first six terms of the sequence satisfying the recurrence equation

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3} \quad , \quad n \geq 3$$

subject to the initial conditions

$$x_0 = 1, x_1 = 2, x_2 = -1 \quad .$$

$X_0 = 1$

$X_1 = 2$

$X_2 = -1$

$X_3 = X_{3-1} + 2X_{3-2} - X_{3-3} = 2$

$X_4 = X_{4-1} + 2X_{4-2} - X_{4-3} = X_3 + 2X_2 - X_1 = -2$

$X_5 = X_{5-1} + 2X_{5-2} - X_{5-3} = X_4 + 2X_3 - X_2 = 3$

$X_6 = X_{6-1} + 2X_{6-2} - X_{6-3} = X_5 + 2X_4 - X_3 = -3$

2. Solve explicitly the recurrence equation

$$x_n = 5x_{n-1} - 6x_{n-2} \quad ,$$

with initial conditions

$$x_0 = 0, x_1 = 1 \quad .$$

$z^2 = 5z^1 - 6z^0$

$z^2 = 5z - 6 \longrightarrow 0 = z^2 - 5z + 6$

$0 = (z-3)(z-2)$

$\left.\begin{array}{l} z_1 = 3 \\ z_2 = 2 \end{array}\right\} X_n = C_1(3)^n + C_2(2)^n$

$\left.\begin{array}{l} X_0 = 0 = C_1 + C_2 \\ X_1 = 1 = 3C_1 + 2C_2 \end{array}\right\} C_1 = -C_2$

$1 = 3(-C_2) + 2C_2$

$1 = -3C_2 + 2C_2$

$1 = -C_2 \rightarrow C_2 = -1 \; , \; C_1 = 1$

$\rightarrow X_n = 3^n - 2^n$

3. (Corrected Sept. 6, 2025, thanks to Caroline Hall [who won a dollar].)

In a certain species of animals, only one-year-old, two-year-old are fertile.

The probabilities of a one-year-old, two-year-old, female to give birth to a new female are p_1, p_2, respectivley.

Assuming that there were c_0 females born at $n = 0$, c_1 females born at $n = 1$ Set up a recurrence that will enable you to find the **expected** number of females born at time n.

In terms of c_0, c_1, p_1, p_2, how many females were born at $n = 4$?

$X_n = P_1 X_{n-1} + P_2 X_{n-2} \; ; \; \underbrace{X_0 = C_0}_{n=0}, \underbrace{X_1 = C_1}_{n=1}$

$X_2 = P_1 X_1 + P_2 X_0 \rightarrow X_2 = P_1 C_1 + P_2 C_0$

$X_3 = P_1 X_2 + P_2 X_1 \rightarrow X_3 = P_1(P_1 C_1 + P_2 C_0) + P_2 C_1$

$X_4 = P_1 X_3 + P_2 X_2 \rightarrow X_4 = P_1[(P_1^2 + P_2)C_1 + P_1 P_2 C_0] + P_2[P_1 C_1 + P_2 C_0]$

$X_4 = (P_1^3 + 2P_1P_2)C_1 + (P_1^2 P_2 + P_2^2)C_0$